Major Field Test in Computer Science
Sample Questions

The following questions illustrate the range of the test in terms of the abilities measured, the disciplines covered, and the difficulty of the questions posed. They should not, however, be considered representative of the entire scope of the test in either content or difficulty. The notation pages and the answer key follow the questions.

1. Consider the following pseudocode fragment, where $\times$ is an integer variable that has been initialized.

\[
\begin{align*}
\text{int } & \quad i \leftarrow 1 \\
\text{int } & \quad j \leftarrow 1 \\
\text{while} \quad & (i < 10) \\
& \quad j \leftarrow j \times i \\
& \quad i \leftarrow i + 1 \\
& \quad \text{if} \quad (i == \times) \\
& \quad \quad \text{break} \\
& \quad \text{end if} \\
\text{end while}
\end{align*}
\]

Which of the following statements is (are) true at the end of the \textbf{while} loop?

- I. $(i == 10)$ or $(i == \times)$
- II. If $\times > 10$, then $i == 10$
- III. If $j == 6$, then $\times == 4$

(A) None
(B) I only
(C) III only
(D) II and III only
(E) I, II, and III
2. In the binary search tree above, finding node $E$ requires one comparison and finding node $A$ requires four comparisons. What is the expected number of comparisons required to find a node chosen at random?

(A) 1.75  
(B) 2  
(C) 2.75  
(D) 3  
(E) 3.25

3. A stack can be implemented with an array $A[0..N-1]$ and a variable pos. The push and pop operations are defined by the following code.

```plaintext
push (x)
    A[pos] ← x
    pos ← pos - 1
end push

pop ( )
    pos ← pos + 1
    return A[pos]
end pop
```

Which of the following will initialize an empty stack with capacity $N$ for this implementation?

(A) pos ← -1  
(B) pos ← 0  
(C) pos ← 1  
(D) pos ← N - 1  
(E) pos ← N
4. A processor with a word-addressable memory has a two-way set-associative cache. A cache line is one word, so a cache entry contains a set of two words. If there are \( M \) words of memory and \( C \) cache entries, how many words of memory map to the same cache entry?

(A) \( \frac{C}{2} \)

(B) \( \frac{M}{2C} \)

(C) \( \frac{M}{C} \)

(D) \( \frac{2M}{C} \)

(E) \( \frac{M}{2} \)

5. Which of the following regular expressions will not generate a string with two consecutive 1s? (Note that \( \varepsilon \) denotes the empty string.)

I. \((1 + \varepsilon)(01 + 0)^*\)

II. \((01 + 10)^*\)

III. \((0 + 1)^*(0 + \varepsilon)\)

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) II and III only
6. Consider the following recursive function.

```c
int Fun ( int n )
if ( n == 4 )
    return 2
else
    return 2 * Fun ( n + 1 )
end if
end Fun
```

What is the value returned by the function call `Fun ( 2 )`?

(A) 2
(B) 4
(C) 8
(D) 16
(E) 24

7. If `A[0..n-1]` is an array with `n` elements and procedure `Swap` exchanges its arguments, then the following code sorts `A` in descending order.

```c
int j ← 0
while ( j < n - 1 )
    int k ← 0
    while ( k < n - j - 1 )
            Swap ( A[k], A[k + 1] )
        end if
        k ← k + 1
    end while
    j ← j + 1
end while
```

If `A` initially contains `n` different elements sorted in ascending order, how many calls to `Swap` are made in total?

(A) `n - 1`
(B) `n`
(C) `n(n - 1)/2`
(D) `(n - 1)(n - 2)`
(E) `n(n - 1)`
8. If $A, B$, and $C$ are Boolean variables, which of the following is (are) true?

I. $A \land (B \lor C) = (A \land B) \lor (A \land C)$

II. $A \lor (B \land C) = (A \lor B) \land (A \lor C)$

III. $(A \land B) \lor C = C \lor (B \land A)$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

9. A personal identification number (PIN) that opens a certain lock consists of a sequence of 3 different digits from 0 through 9, inclusive. How many possible PINs are there?

(A) 120
(B) 360
(C) 720
(D) 729
(E) 1,000
10. Consider the statement “Every connected graph has one or more vertices of degree 2.” Each of the five graphs below has one or more connected components. Which of the five graphs is a counterexample to the statement?

(A) 

(B) 

(C) 

(D) 

(E)
11. Following is a definition of a widget and a declaration of an array A that contains 10 widgets. The sizes of a byte, short, int, and long are 1, 2, 4, and 8 bytes, respectively. Alignment is restricted so that an n-byte field must be located at an address divisible by n. The fields in a struct are not rearranged; padding is used to ensure alignment. All widgets in A must have the same size.

```c
struct widget
    short s
    byte b
    long l
    int i
end widget

widget A[10]
```

Assuming that A is located at a memory address divisible by 8, what is the total size of A, in bytes?

(A) 150
(B) 160
(C) 200
(D) 240
(E) 320

12. Consider a virtual memory system running on a RISC CPU. Page tables are not locked in memory and may be swapped to disk. An lw (load word) instruction reads one data word from memory; the address is the sum of the value in a register and an immediate constant stored in the instruction itself. Neither machine instructions nor page-table entries nor data words can cross a page boundary. In the worst case, how many page faults could be generated as a result of the fetch, decode, and execution of an lw instruction?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6
13. Four processes—$P_1$, $P_2$, $P_3$, and $P_4$—are scheduled to use a single processor. The following table shows the arrival time and duration of each of the four processes.

<table>
<thead>
<tr>
<th>Process</th>
<th>Arrival Time (seconds)</th>
<th>Duration (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>$P_2$</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$P_3$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$P_4$</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

If the system uses a preemptive shortest-job-first scheduling algorithm, what is the average waiting time for the four processes, in seconds?

(A) 2  
(B) 3  
(C) 4  
(D) 5  
(E) 6

14. The figure above represents a nondeterministic finite automaton with accepting state $D$. Which of the following strings does the automaton accept?

(A) 001  
(B) 1101  
(C) 01100  
(D) 000110  
(E) 100100
15. Given that
   
   \[ B(x) \text{ means "} x \text{ is a bear"}, \]
   
   \[ F(x) \text{ means "} x \text{ is a fish"}, \]
   
   \[ E(x, y) \text{ means "} x \text{ eats } y\”, \]

   what is the best English translation of
   
   \[ \forall x [F(x) \rightarrow \forall y (E(y, x) \rightarrow B(y))] \]?

   (A) All fish eat bears.
   (B) Every fish is eaten by some bear.
   (C) Bears eat only fish.
   (D) Every bear eats fish.
   (E) Only bears eat fish.

16. Consider the following table in a relational database.

<table>
<thead>
<tr>
<th>Last Name</th>
<th>Rank</th>
<th>Room</th>
<th>Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith</td>
<td>Manager</td>
<td>234</td>
<td>Morning</td>
</tr>
<tr>
<td>Jones</td>
<td>Custodian</td>
<td>33</td>
<td>Afternoon</td>
</tr>
<tr>
<td>Smith</td>
<td>Custodian</td>
<td>33</td>
<td>Evening</td>
</tr>
<tr>
<td>Doe</td>
<td>Clerical</td>
<td>222</td>
<td>Morning</td>
</tr>
</tbody>
</table>

According to the data shown in the table, which of the following could be a candidate key of the table?

(A) \{Last Name\}

(B) \{Room\}

(C) \{Shift\}

(D) \{Rank, Room\}

(E) \{Room, Shift\}
Major Field Test in Computer Science
Notation, Conventions, and Definitions

In this test, a reading knowledge of modern programming languages is assumed. The following notational conventions and definitions are used.

1. All numbers are assumed to be written in decimal notation unless otherwise indicated.
2. \([ x \) denotes the greatest integer that is less than or equal to \( x \).
3. \([ x \) denotes the least integer that is greater than or equal to \( x \).
4. \( g(n) = O(f(n)) \) denotes “\( g(n) \) has order at most \( f(n) \)” and means that there exist positive constants \( C \) and \( N \) such that \( g(n) \leq Cf(n) \) for all \( n > N \).
   \( g(n) = \Omega(f(n)) \) denotes “\( g(n) \) has order at least \( f(n) \)” and means that there exist positive constants \( C \) and \( N \) such that \( g(n) \geq Cf(n) \) for all \( n > N \).
   \( g(n) = \Theta(f(n)) \) denotes “\( g(n) \) has the same order as \( f(n) \)” and means that there exist positive constants \( C_1, C_2, \) and \( N \) such that \( C_1f(n) \leq g(n) \leq C_2f(n) \) for all \( n > N \).
5. \( \exists \) denotes “there exists”.
   \( \forall \) denotes “for all”.
   \( \rightarrow \) denotes “implies”.
   \( \neg \) denotes “not”; “\( \neg A \)” is also used to mean “\( \neg A \)”.
   \( \lor \) denotes “inclusive or”; \( + \) also denotes “inclusive or”, e.g., \( P + Q \) can denote “\( P \) or \( Q \)”.
   \( \oplus \) denotes “exclusive or”.
   \( \land \) denotes “and”; also, juxtaposition of statements can denote “and”, e.g., \( PQ \) can denote “\( P \) and \( Q \)”.
6. \( \emptyset \) denotes the empty set.

If \( A \) and \( B \) denote sets, then
- \( A \cup B \) is the set of all elements that are in \( A \) or in \( B \) or in both;
- \( A \cap B \) is the set of all elements that are in both \( A \) and \( B \);
- \( A - B \) is the set of all elements in \( A \) that are not in \( B \);
- \( A \subseteq B \) means \( A \) is a subset of \( B \) (that is, any element in \( A \) is also in \( B \));
- \( A \subset B \) means \( A \) is a proper subset of \( B \) (that is, \( A \subseteq B \) and there is at least one element in \( B \) that is not in \( A \));
- \( \bar{A} \) is the set of all elements not in \( A \) that are in some specified universal set; and
- \(|A|\) is the cardinality of \( A \).
7. $\epsilon$ denotes the empty string. If $x$ and $y$ are strings, $x + y$ denotes the set $\{x\} \cup \{y\}$ and $xy$ denotes the concatenation of $x$ and $y$.

If $S$ and $T$ are sets of strings, then

$ST = \{xy \mid x \in S \text{ and } y \in T\}$ denotes the concatenation of $S$ and $T$;

$S + T$ denotes $S \cup T$;

$S^n$ denotes $S S \ldots S$; \(n\) factors

$S^+$ denotes $S \cup S^2 \cup S^3 \cup \ldots$; and

$S^*$ denotes $\{\epsilon\} \cup S^+$.

8. In a grammar, $\alpha \rightarrow \beta$ represents a production rule.

Unless otherwise specified,

(i) symbols appearing on the left-hand side of productions are nonterminal symbols, and the remaining symbols are terminal symbols;

(ii) the leftmost symbol of the first production is the start symbol; and

(iii) the start symbol is permitted to appear on the right-hand side of productions.

9. In a logic diagram

- represents an AND gate;
- represents an inclusive OR gate;
- represents an exclusive OR gate;
- represents a NOT gate;
- represents a NAND gate; and
- represents a NOR gate.

10. In a finite automaton diagram, states are represented by circles, where final (or accepting) states are indicated by two concentric circles. The start state is labeled Start. An arc from state $s$ to state $t$ labeled $a$ indicates a transition from $s$ to $t$ on input $a$.

11. Unless specified otherwise, all code segments are written in pseudocode, where $\leftarrow$ indicates assignment.
Answer Key