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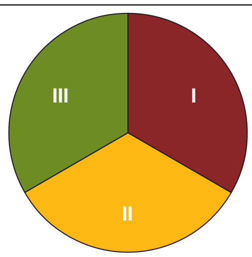
Mathematics: Pedagogy (0065)

Test at a Glance

Test Name	**Mathematics: Pedagogy**
Test Code	**0065**
Time	**1 hour**
Number of Questions	**3 essay questions**
Format	**Essays based on pedagogical questions that focus on planning, implementing, and assessing instruction, calculator allowed**



	Content Categories	Number of Questions	Percentage of Examination
I.	Planning Instruction	1	$33\frac{1}{3}$%
II.	Implementing Instruction	1	$33\frac{1}{3}$%
III.	Assessing Instruction	1	$33\frac{1}{3}$%

About This Test

The Mathematics: Pedagogy test is designed to assess the mathematical knowledge and competencies necessary for a beginning teacher of secondary school mathematics. The test focuses on problem solving, communication, reasoning, and mathematical connections in relation to pedagogy.

This test contains three equally weighted essay questions that cover the following aspects of teaching mathematics in the secondary school: planning—the preparation for teaching; implementation—the presentation of material; and assessment—the evaluation of student understanding.

Following is an overview of the knowledge competencies an examinee can be asked to demonstrate. Answering any question may involve more than one competency.

- Identify and analyze student errors and suggest ways to help the student
- Identify prerequisite skills and understanding for studying a certain topic and explain how you would determine if students have these skills
- For a particular problem, identify several problem-solving strategies that might be useful to students
- Use appropriate forms of representation and a variety of teaching strategies
- Demonstrate an understanding of connections among mathematical topics and real-world situations
- Discuss appropriate use of technology in the context of planning, implementing, or assessing a lesson
- Know how to teach different gender, racial, ethnic, and socioeconomic groups
- Evaluate student learning of mathematics

The level of mathematics used in the pedagogy exercises will not be above that of first-year algebra and may include informal proof, informal geometry, probability and statistics.

Sample Test Questions

This section presents sample questions and constructed-response samples along with the standards used in scoring the exercises. When you read these sample responses, keep in mind that they will be less polished than if they had been developed at home, edited, and carefully presented. Examinees do not know what questions will be asked and must decide, on the spot, how to respond. Readers take these circumstances into account when scoring the responses. Readers will assign scores based on the following scoring guide.

SCORING GUIDE

5

- Responds appropriately and completely to all parts of the question
- Explains how to plan, implement, or assess mathematics instruction in a way that is very likely to achieve the desired goals
- Shows a thorough and strong knowledge of mathematical concepts, theories, facts, procedures, or methodologies relevant to the question
- Where required, demonstrates a strong understanding of how to motivate learners
- Where required, provides strong explanations that are well supported by relevant evidence

4

- Responds appropriately to almost all parts of the question
- Explains how to plan, implement, or assess mathematics instruction in a way that is likely to achieve the desired goals
- Shows an adequate knowledge of mathematical concepts, theories, facts, procedures, or methodologies relevant to the question
- Where required, demonstrates an adequate understanding of how to motivate learners
- Where required, provides adequate explanations that are well supported by relevant evidence

3

- Responds to a major portion of the question
- Explains how to plan, implement, or assess mathematics instruction in a way that is somewhat likely to achieve the desired goals
- Shows a basic knowledge of mathematical concepts, theories, facts, procedures, or methodologies relevant to the question
- Where required, shows some relevant knowledge of how to motivate learners
- Where required, provides some partially supported explanations

2

- Is incomplete, unclear, and very weak in its understanding of both the pedagogy and the mathematics related to the question OR
- Demonstrates some knowledge of the mathematics related to the question, but no understanding of the pedagogy related to the question

1

- Does not respond successfully to any part of the question
- Demonstrates a lack of knowledge or a misunderstanding of the mathematics related to the question
- Demonstrates a lack of understanding of how to motivate learners; of how to plan, implement, or assess mathematics instruction; of how to justify pedagogical decisions

0

- Is blank, almost-blank, or off-topic

Sample Question 1: Planning

SEVENTH-GRADE LESSON PLAN

Student Assignment:

> A square is divided into four congruent rectangles. The perimeter of each of the four congruent rectangles is 25 units. How many units are in the perimeter of the square you started with?

Objectives/Learning Outcomes: Students in a heterogeneously grouped seventh-grade prealgebra class will learn problem-solving strategies using the student assignment shown above as a stimulus.

Your lesson plan should include the following:

(a) The mathematics that the students need to understand or learn in order to solve the problem,

(b) How you would organize the class for instruction and a rationale for that organization, and

(c) The specific strategies you would teach students to use in solving the problem.

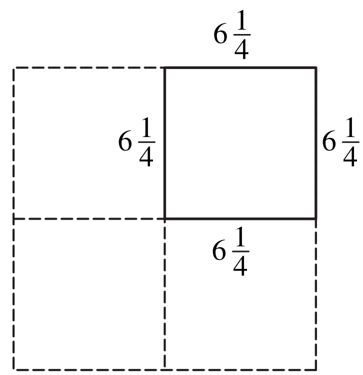
Sample Response That Received a Score of 5:

(a) The students need to understand and be able to apply the terms: square, congruent, perimeter, and rectangle. These terms would be reviewed, with examples prior to the lesson.

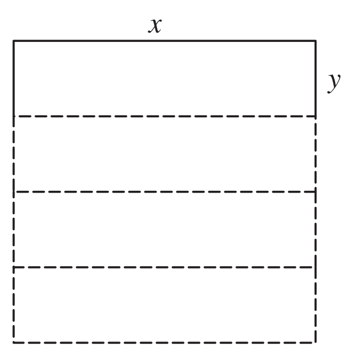
rectangle = quadrilateral with 4 square corners
square = rectangle with all sides equal (congruent)
perimeter = distance around a polygon
congruent polygon = equal in measure and shape

(b) I would have the students work in groups of two. For the discussion of terms, they could agree on and share their definitions and examples.

(c) I would have students use graph paper and draw squares of different perimeters to develop the understanding that perimeter divided by 4 is equal to the length of one side, then find the square with perimeter of 25. I would encourage them to draw diagrams and pictures of ways to divide a square into 4 congruent rectangles, then find, with their diagrams, the perimeter of the original square.



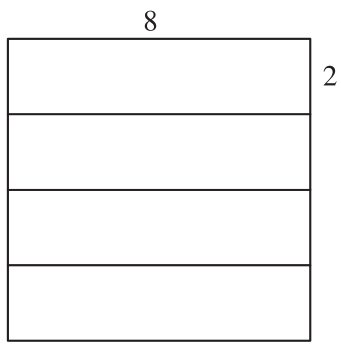
original side = $12\frac{1}{2}$
perimeter = 50



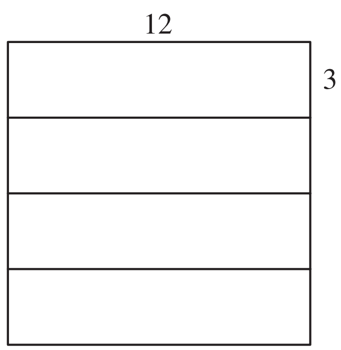
$$2x + 2y = 25$$
$$x = 4y$$
$$8y + 2y + 25$$
$$10y = 25$$
$$y = 2.5$$
$$x = 10$$

original side = 10
perimeter = 40

I would not expect these students to use the equation, but to discover with their diagrams the second solution, possibly by guess and check, for example:



perimeter = 20, close but too low.



perimeter = 30, close but too high

Commentary on Sample Response that Earned a Score of 5

This response received a score of 5 because it clearly demonstrates a strong understanding of the mathematics in the problem, including knowledge of perimeter, congruence, and properties of rectangles and squares, and recognizes that the problem has two possible solutions. It clearly explains how to organize the class for instruction and provides a strong rationale for the organization. The problem-solving strategies are reasonable and clearly explained.

Sample Response That Received a Score of 2:

(a) Concepts students need to know are area of squares and rectangles, perimeter of squares and rectangles, and what squares and rectangles are.

(b) This is a good project for cooperative learning. I would divide the students into small groups and let them work and discover the solution. This teaches students problem solving skills.

(c) Use the following four-step process for problem solving:

1) What is the question
2) What do we know
3) Make a plan
4) Solve and recheck

Commentary on Sample Response that Earned a Score of 2

This response received a score of 2 because it demonstrates a limited understanding of the mathematics involved and does not recognize that the problem has two possible solutions. The response suggests how to organize the class for instruction but provides a weak rationale. Although the response offers some strategies that the students should know or be taught in order to solve the problem, the list of strategies is not complete and the strategies are not carefully explained.

Sample Question 2: Planning

NINTH-GRADE LESSON PLAN

Objectives/Learning Outcomes: Students will develop a conceptual understanding of what absolute value is, how it can be used to represent the distance between two points on a number line, and how it is represented symbolically.

Your lesson plan should include:

(a) How you would motivate the students' interest in the concept of absolute value,

(b) An outline of what you would do to achieve the goals of the lesson, and

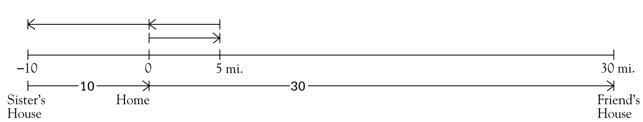
(c) Sample exercises that could be used to reinforce student understanding of the material presented in the lesson.

Sample Response That Received a Score of 5:

I would begin the lesson by talking about driving a car. Pretend you are taking a trip to visit your friend who lives 30 miles away, and that the only route you travel is Highway 441. You leave your house and are headed down the road 5 miles when you discover you left the iron on at home. So you must go back and turn it off. While at home your sister calls and says she wants to come too. But you must drive 10 miles out of your way to pick her up. When you arrive at your friend's house finally, he says "Why are you late? It's only a 30-mile trip?" What do you say?

Using this example I would next encourage my students to find out how many miles the trip was, ask them to come to the board and talk about how they figured this out.

The responses I would be looking for, and would try to lead them to, are



If this problem did not present itself from the student's explanations, I would ask, Why is it 60 mi.? Why don't you add −10 when you pick up the sister? In other words, $5 + (-5) + (-10) + 10 + 30 = ?$

Isn't the sister at −10 on the number line?

How else might this be said so that the reader of your solution knows how you were thinking? This is why mathematicians have developed something called "absolute value."

Absolute value is meant to represent the value of a number, regardless of its sign. So that we could write
$|5| + |-5| + |-10| + |10| + 30 =$
$5 + 5 + 10 + 10 + 30 = 60$ mi.

How else could absolute values help us?

Before I assigned exercises for the students, I would ask them to come up with their own examples and explain to the class. They may need help getting started, so I would offer one more example, and areas they could pursue.

For example: water in a rain gauge
paper in a notebook, etc.

Motivation through challenging them with examples that apply to their lives. Then exercises could be in word problem form, as well as equations, etc.

Commentary on Sample Response that Earned a Score of 5

This response received a score of 5 because it responds appropriately and completely to all parts of the question. The response provides a context to the absolute value problem that may be familiar to the students, demonstrates a strong knowledge of the concept of absolute value, provides an outline that is likely to achieve student understanding of the concept, and provides appropriate exercises to reinforce student understanding.

Sample Response That Received a Score of 2:

Outline

Motivation —

— Start with examples dealing with distance.

— Show that if you went from New York City to Newark, NJ, it would be the same distance from Newark to New York City.

— Relate absolute value to the students' experiences.

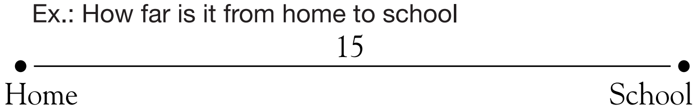
Rationale —

— We study absolute value in mathematics because some measurements (like distance) can only be represented in a magnitude, not a direction.

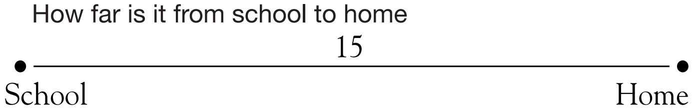
Development —

— Use number line

Ex.: How far is it from home to school



How far is it from school to home



Continue with more examples

Definition of Absolute Value —

— Ask the students to state it in their own words.

Continued Practice —

— Do examples out of book

Commentary on Sample Response that Earned a Score of 2

This response received a score of 2 because it is incomplete and very weak in its understanding of both the concept of absolute value and the teaching of it. The example given to motivate this lesson is appropriate but is not well developed; the lesson plan does not demonstrate an adequate understanding of absolute value and does not outline a plan that is likely to achieve student understanding of the topic; and sample exercises are not listed to demonstrate knowledge of how to assess student understanding.

Sample Question 3: Implementation

Scenario:

You are teaching a unit on probability and statistics in your mathematics class, and you report to your students that a certain candy company claims that the distribution of colors of their candies is $\frac{1}{2}$ orange, $\frac{1}{6}$ brown, and $\frac{1}{3}$ yellow. You hand each student a bag that contains 24 candies, and you ask each student to predict the number of orange, brown, and yellow candies in his or her bag. You then have each student open his or her bag and record the actual number of candies of each color.

(a) Based on the scenario above, identify TWO instructional goals for this lesson.

(b) After the students have recorded their results, describe ONE activity you would have students do next in this lesson.

(c) A simulation is an experiment that models a real-life situation. Describe ONE simulation (using a spinner, a die, or cards) that would model the situation of determining the distribution of the colors of the 24 candies in a bag of candies from this company.

Sample Response That Received a Score of 5:

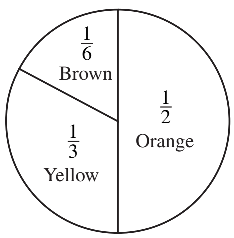
(a) Using candy in the classroom is a motivational factor in and of itself. Determining the claims of the candy manufacturer gives students a real-life situation using statistics. Two instructional goals would be:

1) Predicting outcomes by using percentages
2) Understanding error in probability

For the first goal, students would be able to predict that 12 should be orange, 8 should be yellow, and 4 should be brown. Then they should see if their prediction was correct. This would lead them to understanding error. With a small sample of 24, the predictions will not necessarily be the same. However, if the students take a larger sample size, then the prediction should be closer to the percentage.

(b) The next activity I would have the students do would be to record the class results on a table. This way, students can see that when sample sizes are increased, the percentage of probability is much more accurate. For example, one student might only have 7 orange, another student might have 16 orange. With the class' contributions, the percentage of orange should move closer to one-half or 12 orange per bag.

(c) Using a fair spinner that looks like this



Have the students spin this 24 times and record each result. This simulation should have the same distribution as the bag of candy.

Commentary on Sample Response that Earned a Score of 5

This response received a score of 5 because it shows a clear and strong understanding of statistics, probability, and simulation and of how to teach these topics. Two different instructional goals are identified and the response indicates what the students would do or learn to do in the mathematics during the lesson. The activity furthers the goals of the lesson, helps students learn about probability and statistics, and has students actively doing something with the information collected. The simulation correctly describes how to use a spinner to emulate the distribution of candy colors. The response clearly demonstrates a full understanding of the mathematical content necessary to answer all parts of the question.

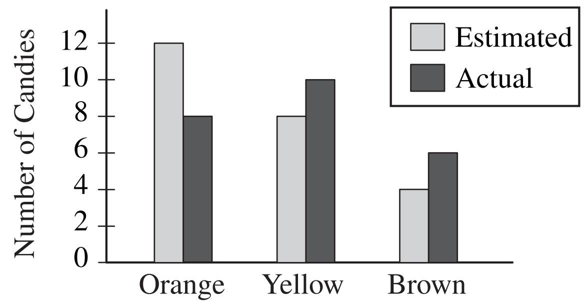
Sample Response That Received a Score of 2:

(a) Two instructional goals for this lesson on probability and statistics would be the following:

1) An objective of this lesson would be to allow students to practice estimation techniques and build on their prior knowledge of statistics

2) An objective of this lesson would also be to compare the estimated results to the actual results graphically (to connect the topics)

(b) After students recorded their results, I would have them graphically illustrate them with a bar chart. Students would represent the quantities as in the following sample bar graph:



The graph shown was based on the statistical information provided by the candy company as well as the example results of a bag of 24 candies containing 8 orange candies, 10 yellow candies, and 6 brown candies.

Students would then do an analysis of whether the candy company's statements were valid or not for their bag of candy. Student results would vary. Have students share their results in groups to compare them. Finally, summarize the results from the class with some overall assumptions.

(c) For a simulation, I would provide groups or pairs of students (depending on class size) with dice. Students would estimate the probability of rolling a 1, 2, 3, 4, 5, or 6. Students could roll the dice 100 times and tally the results as they go. They could compare the actual results to their estimations.

After this introduction to probability and statistics, students may learn to accurately calculate the expected probability for a situation or event.

Commentary on Sample Response that Earned a Score of 2

This response received a score of 2 because it shows a limited understanding of the mathematics and how to teach it. The two instructional goals listed in part (a) are essentially the same. The simulation described in part (c) shows how to estimate events that have a one-sixth probability of occurring. It does not model the situation of determining the color distribution of candies.

Sample Question 4: Implementation

Using the method of elimination of a variable, a student correctly solves systems I and II below, but is unable to solve system III.

(I) $3x + 8y = 12$
$2x + 8y = 9$

(II) $3x + 7y = 5$
$9x - 7y = 3$

(III) $2x + 3y = 1$
$5x + 2y = 1$

(a) Give a likely explanation as to why the student was unsuccessful in solving system III.

(b) Develop a set of approximately 5 sample exercises designed to extend, in a step-by-step fashion, the student's understanding of the method of elimination of a variable so that the student could successfully solve system III.

(c) For each of the sample exercises in your response to part (b), briefly state (i) how the exercise can extend the student's understanding of the method of elimination of a variable and (ii) how the exercise follows naturally from prior exercises and leads naturally to the next exercise.

Sample Response That Received a Score of 5:

(a) The student who successfully solves problems I and II, but not III, probably understands that two equations can be added or subtracted to eliminate a variable. Apparently the student does not understand that one, or both, of the equations may be altered by multiplying both sides of the equation by the same number to create coefficients of one variable that combine up to "0". Furthermore, the student does not understand, or has not thought of, the principle that two quantities equal to the same thing are equal to one another, so that in this case, $2x + 3y = 5x + 2y$, $y = 3x$, and $2x + 3(3x) = 1$, etc.

(b) (1) Recall we can multiply both sides of an equation by 2.
$2(7) + 3(1) = 17$
$4(7) + 6(1) = 34$

(2) Use what you have learned to alter one of the equations, so that the variable y can be eliminated:
$5x + 4y = -3$
$6x - 2y = 10$

(3) Convert the first equation to eliminate the variable x, and solve the equation. Then convert the same equation in another way to eliminate the variable y and solve the equation.
$2x + y = 2$
$14x + 5y = 8$

(4) Convert both equations so that one of the variables can be eliminated. Then solve the equation.
$2x - 3y = 3$
$3x - 2y = 7$

(5) Convert both equations to eliminate a variable.
$2x + 3y = 0$
$6x - 7y = -8$

(c) Exercise (1) demonstrates to the student that when both sides of an equation are multiplied by the same number, the equality relationship, but not the quantity, is preserved.

Exercise (2) asks the student to consider applying knowledge from (1) to altering a single equation with the object of eliminating a particular variable in order to solve the system of equations.

Exercise (3) helps the student to realize that either variable can be eliminated, and that this can be achieved in more than one way.

Exercise (4) shows the student that keeping the solution simple may require altering both equations.

Exercise (5) shows the student that the solution for x and y does not have to be an integer. In this example, $x = -\frac{3}{4}$ and $y = \frac{1}{2}$. Depending on the ability level of the class, further examples yielding an infinite solution set or an empty solution set could be used.

Commentary on Sample Response that Earned a Score of 5

This response received a score of 5 because it responds appropriately and completely to all parts of the question. The response correctly identifies that the student understands the process of adding and subtracting equations to eliminate variables in solving a two-variable system of equations but has trouble understanding how to solve systems by elimination that require more than one step. The response gives a series of 5 sample exercises that appropriately extends and further develops the method of elimination in a step-by-step manner and provides appropriate rationales for the selection and order of the five questions.

Sample Response That Received a Score of 2:

(a) The student can solve problems I and II since a variable is eliminated by adding or subtracting the two equations. In problem III, when the equations are combined by either addition or subtraction a variable is NOT eliminated.

(b)

$x - y = 18$	multiply equation (1) by 2 and add
$x - 2y = 32$	
$2x - 3y = 30$	multiply equation (2) by 3 and subtract
$x - y = 10$	
$3x - 4y = 16$	multiply equation (1) by 3 and
$5x + 3y = 4$	equation (2) by 4 and add

Commentary on Sample Response that Earned a Score of 2

This response received a score of 2 because it is incomplete, unclear, and incorrect in its understanding of both the pedagogy and mathematics of solving systems of equations by elimination. The response correctly identifies why the student is able to solve systems I and II but provides an inadequate or incomplete reason for why the student is unable to solve system III. The series of sample exercises and the rationale for their selection and order is incomplete and incorrect.

Sample Question 5: Assessment

Sample of Student Work
$3^3 \times 3^2 = 3^5$
$2^4 \times 3^4 = 6^8$
$7^3 \times 3^7 = 21^{10}$
$5^3 \div 5^3 = \left(\frac{5}{5}\right)^0 = 1$
$7^3 \div 14^4 = \left(\frac{1}{2}\right)^{-1}$

(a) Identify one concept the student work above reveals that the student understands and one concept the student appears to have difficulty understanding.

(b) Describe an instructional strategy that will help the student understand the difficult concept identified in part (a).

(c) Write three problems to assess student understanding of exponentiation after reteaching and explain what concept each problem will assess.

Sample Response That Received a Score of 5:

(a) The student understands that you add exponents when you are multiplying numbers and subtract exponents when you are dividing numbers. However, the student seems to have missed the concept that the base numbers must be equal for you to add or subtract the exponents.

(b) The instructional strategy that would help the student would be to write out both sides of the equation, for example, does $2^4 \times 3^4 = 6^8$?

Expanding the left-hand side:

$2^4 \times 3^4 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3.$

Expanding the right-hand side:

$$\begin{aligned} 6^8 &= 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \\ &= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \\ &\quad \times (2 \times 3) \times (2 \times 3) \times (2 \times 3). \end{aligned}$$

Then ask how many 2's and 3's are on each side. There are four 2's and four 3's on the left-hand side and there are six 2's and six 3's on the right-hand side. Thus the student shows that $2^4 \times 3^4 \neq 6^8$. Having written out an equation as above, ask how the right-hand side could be rewritten so that it would be equal. If no ideas arise, ask the student if 6^4 is a possibility, allowing plenty of time to think and several examples similar to this, including examples of division problems. This should clarify the concept of exponents and when they can be added or subtracted and where the base numbers should be multiplied or divided instead and the exponents are left alone.

(c) Three problems to assess student understanding of exponentiation:

1) $3^2 \times 4^2 = 12^2$ This will reveal if the student understands that the base numbers should be multiplied and the exponents left alone when the bases are different but the exponents are the same.

2) $5^2 \times 5^7 = 5^9$ This will reveal if the student understands that the exponents can be added when the base numbers are the same.

3) $7^2 \times 3^4 = 63^2$ This will reveal if the student is able to synthesize the two concepts: first, rewriting 3^4 with $3^2 \times 3^2$ so that the exponents will match that of 7^2; and second, combining the bases and leaving the common exponent to get 63^2. The student can demonstrate an understanding of these concepts by any of the following ways:

$7^2 \times 3^2 \times 3^2 = (7 \times 3 \times 3)^2$ or

$\left(7^2 \times 9^2\right)$ or $\left(21^2 \times 3^2\right) = 63^2$.

Commentary on Sample Response that Earned a Score of 5

This response received a score of 5 because it demonstrates a strong understanding of content and pedagogical knowledge. The response correctly identifies one concept the student does and one the student does not understand. The response describes an appropriate instructional strategy and details on how to use the strategy to help develop student understanding. The three problems presented are accompanied by explanations as to what they assess. The response specifically addresses how to work with different bases with the same exponent.

Sample Response That Received a Score of 3:

(a) $3^3 \times 3^2 = 3^5$ — understood

$2^4 \times 3^4 = 6^8$ — not understood

(b) If you are multiplying the same number taken to an exponent, add the exponents. But if you are multiplying numbers and the exponent is the same, only multiply the numbers and leave the exponent alone. For example, if you have $2^3 \times 3^3$, that is the same as saying $2 \times 2 \times 2 \times 3 \times 3 \times 3$. If you multiply the 2's and 3's together you would get $6 \times 6 \times 6$ or 6^3.

(c) 1) $5^5 \times 5^2 =$ This problem verifies that the student understands that you add exponents when the root is the same.

2) $5^7 \times 3^7 =$ This problem verifies that the student understands that you multiply roots and leave exponents the same.

3) $5^7 \times 3^6 =$ This problem verifies that the student is able to make the next logical step after understanding how to deal with problems with the same exponents and different bases.

Commentary on Sample Response that Earned a Score of 3

This response received a score of 3 because it shows a basic knowledge of the mathematical content and describes how to assess the mathematical instruction in a way that is somewhat likely to achieve student understanding. The description of what the student understands in part (a) and the instructional strategy to help develop student understanding in part (b) are not clearly presented and lack detail.

Sample Question 6: Assessment

Scenario: Students in an Algebra I class were given the following problem on an end-of-unit test. The student problem is to be used in answering parts (a) and (b) of this question.

The Student Problem: Using what you have learned in this unit, solve each part of the following problem, showing all your work. Your response will be scored on the correctness of its mathematics as well as on how well it is presented and communicates mathematical reasoning and problem-solving strategies.

(i) Graph and label the points $O(0, 0), A(-3, -4), B(3, 4),$ and $C(4, -3)$ on the xy-coordinate plane.

(ii) Write an equation for the line containing points A and B.

(iii) Show that lines $\overleftrightarrow{OC}$ and $\overleftrightarrow{AB}$ are perpendicular.

(iv) Points A, B, and C determine a circle with center O. Identify two points in the second quadrant that are also on this circle.

(v) What is the circumference of the circle?

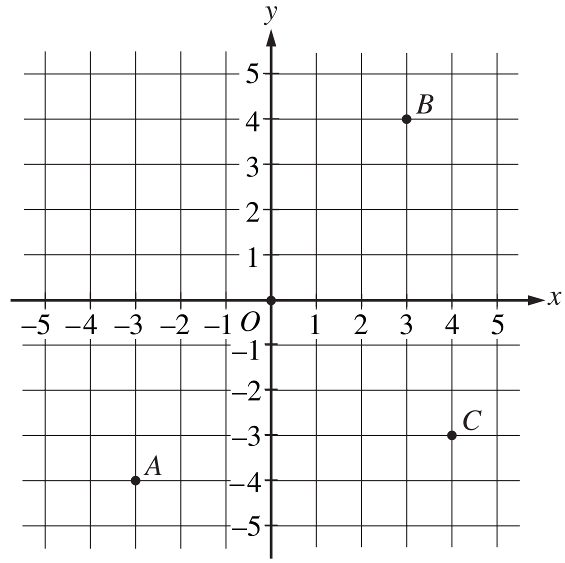
Your Task:

(a) Write a solution to each part of the student problem above, showing your work.

(b) For each part of the student problem, identify the mathematical knowledge, skills and abilities it is designed to assess.

Sample Response That Received a Score of 5:

(i)



Math knowledge: drawing x- and y-axes in a coordinate plane and labeling ordered pairs on the coordinate plane

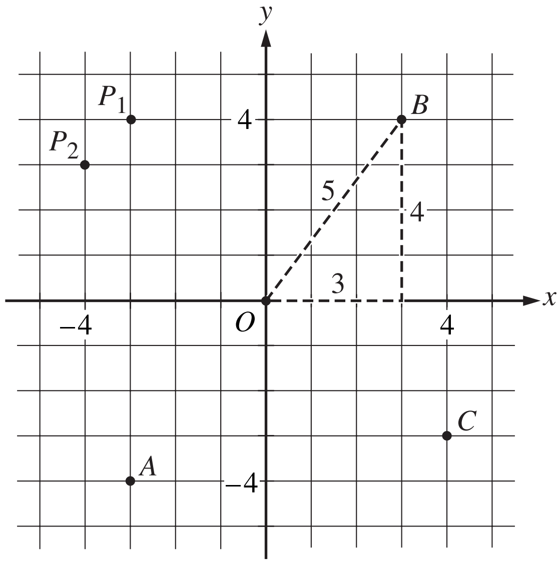
(ii) $m = \frac{\Delta y}{\Delta x} = \frac{4-(-4)}{3-(-3)} = \frac{8}{6} = \frac{4}{3}$, using the slope-intercept form $y = mx + b$, substitute to get $-4 = \frac{4}{3}(-3) + b = -4 + b$, therefore $b = 0$ and $y = \frac{4}{3}x$.

Math knowledge: knowing what slope is and how to derive it from 2 points, what the y-intercept is and how to derive it given 2 points, how to use the slope-intercept form of a line, and using algebraic skills to solve for a variable

(iii) Show lines $\overleftrightarrow{OC}$ and $\overleftrightarrow{AB}$ are perpendicular by showing their slopes to be negative inverses of each other. Slope of line $\overleftrightarrow{AB}$ is $\frac{3}{4}$ from part (i); slope of line $\overleftrightarrow{OC}$ is $\frac{4-0}{-3-0} = \frac{4}{-3}$.

Math knowledge: knowing the definition of perpendicular, slope and perpendicularity, and ability to find slope given two points

(iv)



$r^2 = 3^2 + 4^2 = 9 + 16 = 25$, therefore $r = 5$ (negative root not used for distance).

P_1 is $(-3, 4)$ and P_2 is $(-4, 3)$.

Math knowledge: knowing the definition of quadrants, Pythagorean theorem, and calculating and using the length of the radius to find other points on a circle

(v) Circumference $= 2\pi r = 2\pi(5) = 10\pi$.

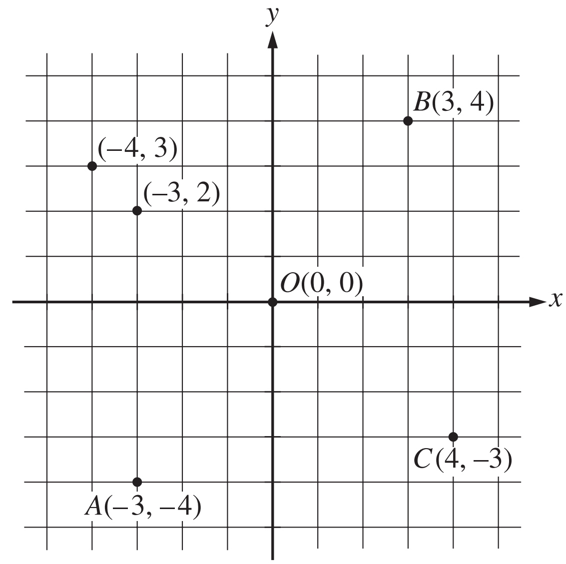
Math knowledge: knowing and applying the formula for circumference of a circle, and using the circumference formula and substituting previously found values

Commentary on Sample Response that Earned a Score of 5

This response received a score of 5 because all parts are answered correctly, clearly, and completely. The solution for the student problem is correct and complete for all five parts and correctly identifies the most significant knowledge, skills, and abilities needed to answer the student problem.

Sample Response That Received a Score of 4:

(i)



This problem assesses the student's ability to plot points on the *xy*-plane. To plot these points, the student must be able to differentiate between the *x*- and *y*-axes and between the *x* and *y* coordinates in an ordered pair. Then the student must know to count in the appropriate direction (positive or negative) to find the points on the appropriate axis.

(ii) slope $= \frac{\text{Rise}}{\text{Run}} = \frac{8}{6} = \frac{4}{3}$ and $y = mx + b$,
$4 = \frac{4}{3}(3) + b = 4 + b$, and $b = 0$.

Equation of the line: $y = \frac{4}{3}x$.

This problem requires the student to find slope. By this level, they should know that they can count the number of units along the y-axis, then along the x-axis to get from point A to point B on the graph.

When they divide $\frac{\text{rise}}{\text{run}}$, they will find the slope. By plugging slope and the coordinates of one point into the slope-intercept form for the equation of a line, they will find b, or the *y*-intercept. They could also use a ruler to draw the line from *A* to *B*, and they will notice that the line intercepts the *y*-axis at zero.

Once they have the slope and *y*-intercept, they can plug them into $y = mx + b$, and they will have the line $y = \frac{4}{3}x$.

(iii) Slope of $\overleftrightarrow{AB}$ is $\frac{4}{3}$; slope of $\overleftrightarrow{OC}$ is $-\frac{3}{4}$, slopes are opposite reciprocals, so they are perpendicular.

The students must know that two perpendicular lines have opposite reciprocal slopes. (Note: They also must know the meaning of "reciprocal.")

(iv) $(-4, 3)$, $(-3, 2)$

For this problem, the students must be able to identify the second quadrant. Then they can use the slope of line $\overleftrightarrow{OC}$ to connect with two more points along that line in the second quadrant.

(v) $C = 2\pi r$

Students must know the formula for the circumference of a circle, and they must identify the circle's radius.

Commentary on Sample Response that Earned a Score of 4

This response received a score of 4 because it responds appropriately to almost all parts of the question. The response gives clear explanations of the knowledge assessed and shows an understanding of the mathematics, although in part (iv) it has a minor error (the second point is incorrect) and does not complete part (v) of the student problem.

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