BENCHMARK MA.6.A.1.1

Reporting Category  Number and Operations

Standard  
**Big Idea 1**  Develop an understanding of and fluency with multiplication and division of fractions and decimals.

Benchmark  
**MA.6.A.1.1**  Explain and justify procedures for multiplying and dividing fractions and decimals.

Item Type  
At Grade 6, this benchmark will be assessed using MC items.

Benchmark Clarification  
Students may identify procedures for multiplying or dividing fractions and/or decimals in the context of expressions, equations, or real-world situations.

Students will choose the correct graphic representation of multiplication or division problems involving fractions or decimals.

Content Limits  
Items may include mixed numbers, fractions, and/or decimals.

Items may include decimals through the hundredths place.

Denominators of fractions used must be less than or equal to 16.

Items will not require the student to simplify fractions.

Items may not include a combination of fractions and decimals.

Stimulus Attributes  
Items should be set in a real-world or mathematical context.

Graphical representations of fractions, mixed numbers, and/or decimals may be used, as appropriate.

Prior Knowledge  
Items may require the student to apply mathematical knowledge described in the Standards from lower grades. This benchmark requires prerequisite knowledge from MA.4.A.2.3 and MA.5.A.6.1.
### BIG IDEA 1: Develop an understanding of and fluency with multiplication and division of fractions and decimals.

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<tr>
<th>BENCHMARK CODE</th>
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<tbody>
<tr>
<td>MA.6.A.1.1</td>
<td>Explain and justify procedures for multiplying and dividing fractions and decimals.</td>
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<tr>
<td>MA.6.A.1.2</td>
<td>Multiply and divide fractions and decimals efficiently.</td>
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<tr>
<td>MA.6.A.1.3</td>
<td>Solve real-world problems involving multiplication and division of fractions and decimals.</td>
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### Access Points for Students with Significant Cognitive Disabilities

#### Independent:
- MA.6.A.1.In.a Express and represent fractions, including halves, fourths, thirds, and eighths, using number names and numerals.
- MA.6.A.1.In.b Solve real-world problems involving fractions, including halves, fourths, thirds, and eighths.
- MA.6.A.1.In.c Identify multiplication as repeated addition of equal groups and multiply one-digit numbers using physical and visual models with numerals.
- MA.6.A.1.In.d Identify division as repeated subtraction of equal groups and divide one-digit numbers using physical and visual models with numerals.

#### Supported:
- MA.6.A.1.Su.a Identify fractions as part of a whole or part of a set.
- MA.6.A.1.Su.b Use fractions, including halves, to solve real-world problems.
- MA.6.A.1.Su.c Use skip counting by 5s and 10s to 50.
- MA.6.A.1.Su.d Solve problems that involve combining (multiplying) equal sets with quantities to 30 using objects and pictures with numerals.
- MA.6.A.1.Su.e Use counting and grouping to separate (divide) quantities to 30 into sets with the same number using objects or pictures.

#### Participatory:
- MA.6.A.1.Pa.b Communicate desire for familiar items to be added or removed in daily activities.
- MA.6.A.1.Pa.c Communicate desire for more and none in two or more daily activities in different parts of the learning environment.
- MA.6.A.1.Pa.d Communicate desire for less in two or more routines or familiar activities.
MA.6.A.1.1: Explain and justify procedures for multiplying and dividing fractions and decimals.

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MA.6.S.6.1 – VERTICAL ALIGNMENT

4th Grade:

**MA.4.A.2.3** Relate equivalent fractions and decimals with and without models, including locations on a number line.

**MA.4.A.6.2** Use models to represent division as:
- the inverse of multiplication
- as partitioning
- as successive subtraction

5th Grade:

**MA.5.A.6.1** Identify and relate prime and composite numbers, factors and multiples within the context of fractions.

**MA.5.A.2.1** Represent addition and subtraction of decimals and fractions with like and unlike denominators using models, place value or properties.

MA.6.A.1.1: Explain and justify procedures for multiplying and dividing fractions and decimals.

7th Grade:

**MA.7.A.3.2** Add, subtract, multiply, and divide integers, fractions, and terminating decimals, and perform exponential operations with rational bases and whole number exponents including solving problems in everyday contexts.

**MA.7.A.3.3** Formulate and use different strategies to solve one-step and two-step linear equations, including equations with rational coefficients.
CONTENT DESCRIPTION: MA.6.A.1.1
Explain and justify procedures for multiplying and dividing fractions and decimals.

Big Idea #1 is to develop an understanding of and fluency with multiplication and division of fractions. MA.6.A.1.1 supports this big idea.

Students may identify procedures for multiplying or dividing fractions and/or decimals in the context of expressions, equations, or real-world situations.

Students will choose the correct graphic representation of multiplication or division problems involving fractions or decimals.

The FCAT is expected to assess MA.6.A.1.1 with multiple choice questions.

It is suggested that teachers read “10 Practical Tips for Making Fractions Come Alive and Make Sense” by Doug M. Clarke, Anne Roche and Annie Mitchell. It was published in Mathematics Teaching in the Middle School, Vol. 13 No. 7, March 2008.

Another NCTM article that is interesting to read is, “What Do Students Need to Learn About Division of Fractions” by Yeping Li, Mathematics Teaching in the Middle School, Vol. 13, No 9, May 2008. This article showcases a number of algorithms for dividing fractions and explains the algebraic reasons that make the algorithms work.

Although it can be efficient for students to automatically use a common algorithm for multiplying and dividing fractions and decimals, this isolated knowledge is not sufficient to verify that students truly understand the process. This is evident when students apply an algorithm, come up with an outlandish answer and do not even question its validity!

While it is still useful to “invert and multiply” when dividing fractions and to “move the decimal” when dividing decimals, the goal of this benchmark is to give students a deeper knowledge of the two operations so that it is not necessary to rely on recalling the algorithms in order to come up with a reasonable solution.

Students and teachers may have used “partitive” and “quotitive” to describe division of whole numbers. Formalizing the difference between the two concepts may give students a reasoning tool to use when dividing fractions and decimals.

Division is called PARTITIVE when you know the number of groups and want to find the number in each group. Example: 124 students must be seated in three school buses. How many students will be in each bus?
Division is called QUOTITIVE when you know the number in each group and want to find the number of groups. Example: 124 students must be assigned to school buses. If 41 students can sit in each bus, how many buses will be needed?

PARTITIVE division with fractions could be explained using this example: A 14½-foot board must be cut into three equal pieces of shelving. How long is each piece of shelving?

QUOTATIVE division with decimals could be explained using this example: A group of people were eating at a restaurant and decided to split the bill. The bill, plus tip came to $145.25. If each person paid $20.75, how many people were in the group?

The links below are interesting exercises for teachers who want to deepen their understanding of partitive and quotative division with fractions:

http://math.arizona.edu/~ime/2006-07/0301_workshop_VA_handouts/Using%20Arrays%20to%20Reason%20with%20Fractions1.doc

http://arapaho.nsuok.edu/~wyattw/webfiles/divpartitivemodel.htm
GLOSSARY OF RELATED MATHEMATICAL LANGUAGE

algorithm  An explicit step-by-step procedure for performing a mathematical computation or for solving a mathematical problem.

common denominator  A whole number greater than zero that is a common multiple of each denominator in two or more fractions (e.g., common denominators for $\frac{1}{6}$ and $\frac{3}{8}$ are 24, 36, 48, ...).

decimal fraction  A fraction using a decimal point to represent the denominator of 10 or a power of 10.

decimal number  A fractional number written using base ten notation; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number).

denominator  The quantity below the line in a fraction. It represents the number of equal parts into which the whole is divided.

dividend  A number to be divided by another number (divisor).

divisible  If a number has a whole number answer when divided by a second number, the first number is divisible by the second number; $x$ is divisible by $y$ if and only if $x = qy$ where $y$ is a whole number.

division  A mathematical operation involving two numbers that tells how many groups there are or how many are in each group.

divisor  The number by which the dividend is divided.

equivalent fractions  Two or more fractions that have the same quotient or that name the same region, part of a set, or part of a segment (e.g., $\frac{1}{3} = \frac{3}{9}$).

expression  A mathematical representation containing numbers, variables, and operation symbols; an expression does not include an equality or inequality symbol.

fraction  A number that represents part of a whole, part of a set, or a quotient in the form $\frac{a}{b}$ which can be read as $a$ divided by $b$.

hundredths place  The place value located two places to the right of the decimal point in a number; one out of a hundred equal parts of a whole.

improper fraction  A fraction whose numerator is greater than its denominator.
least common denominator (LCD)  The smallest common multiple of two given denominators (e.g., the LCD of $\frac{1}{3}$ and $\frac{1}{8}$ is 24).

logical reasoning  The process of using a rational, systematic series of steps based on sound mathematical procedures to arrive at a conclusion; the drawing of conclusions from given facts and mathematical principles; often used as a problem solving strategy.

mixed number  A number composed of an integer and a proper fraction (e.g., $3\frac{2}{9}$).

multiple representations  Various ways to present, interpret, communicate, and connect mathematical information and relationships.

nonrepeating decimal  A decimal that does not repeat; it either terminates or continues in no discernible pattern.

number line  A line on which each point represents a real number.

Example:

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

numerator  The top number in a fraction; it tells the number of equal parts (numerator) out of the total number of parts (denominator) being described by the fraction.

product  The number that is obtained when two or more factors are multiplied.

quotient  The answer to the division of two numbers.

rational number  Any number that can be expressed as a ratio in the form $\frac{a}{b}$ where $a$ and $b$ are integers and $b \neq 0$.

reasonable estimate  An approximation of the result of a given problem or calculation using rational, logical procedures (e.g., rounding).

reasonableness  The justification that a particular solution to a problem is within logical estimates.

repeated subtraction  Subtraction of equal groups from a number; a model or alternative algorithm for division.

repeating decimal  A decimal in which one or more digits repeat infinitely (e.g., 0.3333… or 0.\overline{3}, 5.272727…, or 5.\overline{27}).
**simplify fractions**  To rename fractions to lowest terms by dividing the numerator and denominator by the greatest common factor of the numerator and denominator.

**terminating decimal**  A decimal whose digits do not repeat; all terminating decimals are rational numbers (e.g., 0.7355).

**unit fraction**  A fraction with a numerator of 1.

**unlike denominators**  Two or more fractions with unequal denominators (e.g., \(\frac{6}{17}\) and \(\frac{3}{7}\)).
LITERACY STRATEGIES

An anticipation guide can be used before the lesson to get students thinking about what they may or may not know about the topic. A sample is shown.

**Anticipation Guide – Multiplying & Dividing Fractions**

Directions: In front of each number, write a check in one of the columns. As you learn more about multiplying and dividing fractions, you may go back and change your check and rewrite the incorrect statements to make them true.

<table>
<thead>
<tr>
<th>TRUE</th>
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NUMERACY STRATEGIES

Numeracy, or quantitative literacy, is the ability to reason with numbers and other mathematical concepts. Numeracy includes a comfort with using logic and reasoning and the ability to apply mathematical knowledge to the solution of real-world problems and everyday tasks.

Students can easily learn to find the product of a unit fraction and a whole number by dividing. For example, $1/5 \times 30 = 6$ and students quickly figure out that the same result can be found by performing the easy division of $40 \div 5 = 6$.

Extend this idea to multiplying a larger fraction by a whole number, say $4/5 \times 30$. First the student finds $1/5$ of $30$ and know that the answer is 6. The reasoning continues that $4/5$ is four times larger than $1/5$ and so, $4/5$ of $30$ must be $4 \times 6$ or $24$. 
BRAIN-COMPATIBLE LEARNING STRATEGIES

Constructivists believe that students are the constructors of their own knowledge. Give students plenty of time to examine multiplication and division of fractions in model form, graphic form and in real-world problems before emphasizing any algorithm. Students who construct their own understanding of these two operations will more easily transfer their learning to the mastery of computation strategies and will be able to judge the reasonableness of solutions.
<table>
<thead>
<tr>
<th>QUADRANT C (assimilation)</th>
<th>QUADRANT D (adaptation)</th>
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<tbody>
<tr>
<td>1. Explain the pattern of quotients and products for fractions and decimals.</td>
<td>1. Read about philosopher's reactions to Zeno's dilemmas and create a presentation to share the information with others.</td>
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</tbody>
</table>
| 1. Use an algorithm for fraction multiplication or division to solve a problem.  
2. Fill in the table values with patterns of multiplication and division. | 1. Multiply fractions in the context of Zeno's Paradox problems.  
2. Explain the situations where multiplication results in a product smaller than one of the factors.  
3. Write an original real-world problem that requires multiplication or division of fractions.  
4. Draw a picture that illustrates multiplication or division of fractions in a real-world situation. |

| QUADRANT A (acquisition) | QUADRANT B (application) |
Subject(s) Mathematics

Grade Level 6

Instructional Focus

Number Operation and Concepts:
Students use number, number sense, and number relationships in a problem-solving situation. Students communicate the reasoning used in solving these problems.

Problem-Solving and Mathematical Reasoning: Students apply a variety of problem-solving strategies to investigate and solve problems from across the curriculum as well as from practical applications.

Student Learning

- Students will multiply whole numbers by fractions and explain why the product is less than the whole number.
- Students will explain why division of the whole number by the fraction is an incorrect process to use.
- Students will recognize that the word “of” following a fraction indicates multiplication.

Performance Task

Zeno’s Paradox framed the concept of motion as a dilemma. He described a race from a beginning to an end in terms of finishing sections of the race. He said that first a runner would have to cover half of the distance, next half of the remaining distance, and so on. He suggested that the runner could never finish the race!

Zeno’s Paradox is a story that can capture the attention and imagination of 6th graders. Students should be asked to act out the paradox, informally at first, and then by using real measurement tools such as tape measures and chalk marks on the floor. Plastic “men” could be used as desktop models and the scenario could be acted out and measured there.

A class discussion should ensue that examines the operations needed to calculate ½ of the remaining distance. Some students may think at first that division should be the operation to use. Teachers should steer the
discussion so that multiplication and division by \( \frac{1}{2} \) are examined and contrasted. Students should make journal entries to describe the differences between using the two operations.

Expand on Zeno’s Paradox by changing the fractional distance that is covered each time. Make sure that students make a labeled diagram of the resulting race. Continue the class discussion and observation of using multiplication and division with the new fraction.

Student journals should include written responses to these prompts:
- How can you multiply 50 by a number and end up with a product that is smaller than 50?
- How can you divide 50 by a number and end up with a product that is larger than 50?
Illustrations should accompany the written explanations.

**Scoring Guide**

To check for student understanding, ask students to answer the following questions and to write out the explanation:

1. For an experiment, a 20-ml test-tube is being filled with water from a shallow puddle. An eye-dropper is used to collect the water from the puddle and deposit it into the test-tube. The eye-dropper picks up an average of \( \frac{3}{4} \) ml of water each time. How many times will it take to fill the test-tube?

2. A bottle contains 18\(\frac{1}{2}\) ounces of juice. John drank \(\frac{2}{3}\) of the juice. How many ounces did he drink?

**Attachments/Resources**

Worksheet: “Zeno”
Zeno’s paradox

Zeno of Elea was a Greek philosopher who was born around 495 B.C. He is famous for posing questions called “paradoxes” and challenging mathematicians to disprove them. Zeno’s most famous paradox was about motion. He made the argument that to get from the beginning of a race to the end, the runner must first cover half the distance. Then, to get from that midway point to the end, the runner must cover half of the remaining distance. Then the runner must cover half of the remaining distance and so on. The runner can never get to the end of the race!

Write on the diagram to illustrate the runner doing what Zeno describes in a 100 meter race:

1. What is $\frac{1}{2}$ of 100? _______ What operation must you use to find $\frac{1}{2}$ of 100? _______

What is $\frac{1}{2}$ of 50? _______ What operation must you use to find $\frac{1}{2}$ of 50? _______

Explain how solving $\frac{1}{2}$ of 100 gives you the same answer as solving 100 ÷ 2.
2. Whenever you see the word “of” following a fraction, it means that you have to multiply.

a) Find \( \frac{1}{8} \) of 80. _____________________

b) Find \( \frac{1}{10} \) of 50. _____________________

c) Find \( \frac{2}{3} \) of 12. _____________________

d) Find \( \frac{3}{5} \) of 25. _____________________

Let’s change Zeno’s Paradox a little bit. Instead of covering \( \frac{1}{2} \) the distance each time, let’s say that the runner covers \( \frac{3}{4} \) of the remaining distance each time. Then what would happen?
Write on the diagram to illustrate the runner doing this in a 160 meter race:

3. What is \( \frac{3}{4} \) of 160? ______________ Write a multiplication problem for this: ______________

4. What is \( \frac{3}{4} \) of 40? ______________ Write a multiplication problem for this: ______________

5. Does multiplication always result in a product that is larger than what was multiplied? _____
Write a few sentences to explain your answer:
6. What would happen if you tried to use $160 \div \frac{3}{4}$ to find the distance that the runner covers?

Find the quotient: $160 \div \frac{3}{4} = \underline{\phantom{00000}}$ (You should get a pretty big number!)

Write a word problem that could be solved using 160 meters divided by $\frac{3}{4}$. 


### DRAW A PICTURE

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<tr>
<th>Subject(s)</th>
<th>Rigor/Relevance Framework</th>
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<tbody>
<tr>
<td>Mathematics</td>
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<table>
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<tr>
<th>Grade Level</th>
<th>Instructional Focus</th>
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<tr>
<td>6</td>
<td>Number Operation and Concepts: Students use number, number sense, and number relationships in a problem-solving situation. Students communicate the reasoning used in solving these problems.</td>
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<td>Problem-Solving and Mathematical Reasoning: Students apply a variety of problem-solving strategies to investigate and solve problems from across the curriculum as well as from practical applications.</td>
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<tr>
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<td>Student Learning: The student will find the quotient of two fractions presented in a real-world situation.</td>
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<td>Worksheet: Draw a Picture</td>
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EXAMPLE: A scout troop is getting ready to go on a day hike. The leader has a bag filled with 3 cups of peanuts that the scouts will share. The leader uses a measuring cup to give ¼ cup to each scout. How many scouts will get peanuts?

The picture shows three cups of peanuts. I decided to split each cup into 4 sections.

That means that 12 scouts will get peanuts.

The math problem is $3 \div \frac{1}{4} = 12$.

1. A department store is offering free wrapping for the holidays and to get ready for the rush of customers, the wrapping department decides to pre-cut the paper.
A roll of paper contains 4 ½ yards of paper. The roll of paper is cut into ¾ - foot pieces. How many pieces can be cut from each roll?

2. A carpenter is making shelves from a 12-foot piece of lumber. Each shelf is 2 ½ feet long. How many shelves can be made and how much lumber will be left over?

3. A ball of yarn contains 18 meters of yarn. To make fringe for a scarf, the yarn needs to be cut into pieces that are each 2/3 of a meter in length. How many pieces of fringe can be cut from the ball of yarn?

4. Jay’s grandma made a 7½ -cup pot of spaghetti sauce and she plans to divide it into 1⅔ -cup servings and freeze them. How many servings can she make? How much will be left over?

5. Steven is installing tile in a small room. Each row requires 5 ¼ tiles. He bought 210 tiles. How many rows can he cover with tiles?
6. A bookshelf is 4 2/3 feet long. The purpose of the shelf is to hold math books that are each 1/6 of a foot thick. How many math books can fit on the shelf?

7. 

a) Make up a problem that involves real-world situations and the division of fractions. Write it out.

b) Draw an illustration of your problem.

c) Solve your problem. Show all work.
GOLD SEAL LESSON

GOLD SEAL LESSON

PATTERN INVESTIGATION

Subject(s)
Mathematics

Rigor/Relevance Framework

Grade Level
6

Instructional Focus

Number Operation and Concepts: Students use number, number sense, and number relationships in a problem-solving situation. Students communicate the reasoning used in solving these problems.

Problem-Solving and Mathematical Reasoning: Students apply a variety of problem-solving strategies to investigate and solve problems from across the curriculum as well as from practical applications.

Student Learning

- Students will predict the outcome when a number is divided by a number between 0 and 1.
- Students will predict the outcome when a number is multiplied by a number between 0 and 1.

Performance Task

This lesson provides a guided exploration of what happens to quotients and products when a number is multiplied or divided by a value between zero and one.

The investigation begins with a list of similar problems in spreadsheet form. The same number is multiplied or divided by 4, 3, 2, 1, then various fractional values and finally 0. Students compute the product or quotient. This is easy for them to do when the divisor or factor is 1 or greater. As they work their way down to the fractions, students should be encouraged to observe the way the products have decreased and the quotients have increased. It is logical that this pattern of decreasing or increasing will continue when the numbers are fractions.

Another piece of the investigation asks students to consider a geometric shape and to perform the multiplication or division on that shape. This provides a visual illustration of the pattern that may click with certain learners.

After each investigation, students are asked to record their
observations and to formulate “rules” for multiplying and dividing by numbers between 0 and 1.

**Scoring Guide**

Check for student understanding by giving a quick quiz that focuses on the two objectives outlined in the lesson. The teacher can then plan additional instruction for students who do not demonstrate understanding.

A multiple choice format could work well. A sample question is shown below:

Use your knowledge of fractions to choose the correct product of $\frac{5}{7} \times 20$ without multiplying:

- a) 28
- b) 26.5
- c) $21\frac{1}{7}$
- d) $14\frac{2}{7}$

**Attachments/Resources**

Worksheet: Pattern Investigation
# PATTERN INVESTIGATION

1. Fill in the table and look for patterns.

<table>
<thead>
<tr>
<th>multiplication</th>
<th>product</th>
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<tbody>
<tr>
<td>6 x 60 =</td>
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<tr>
<td>5 x 60 =</td>
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<tr>
<td>4 x 60 =</td>
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<td>3 x 60 =</td>
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<td>2 x 60 =</td>
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<td>1/15 x 60 =</td>
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<td>1/20 x 60 =</td>
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<tr>
<td>1/30 x 60 =</td>
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<tr>
<td>0 x 60 =</td>
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</tbody>
</table>

You kept multiplying 60 by numbers that kept getting smaller and smaller. Write a description of what happened to the products.

When you multiply two numbers, is the product always a larger number than the two factors? Explain.

2. What happens to the size of the product as you multiply by numbers between 1 and 0? Explain!

<table>
<thead>
<tr>
<th>multiplication</th>
<th>product</th>
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</thead>
<tbody>
<tr>
<td>3 x 60 =</td>
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<tr>
<td>2 x 60 =</td>
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<tr>
<td>1 x 60 =</td>
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<td>0.5 x 60 =</td>
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<td>0.4 x 60 =</td>
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<td>0.3 x 60 =</td>
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<td>0.2 x 60 =</td>
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<tr>
<td>0.1 x 60 =</td>
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<tr>
<td>0 x 60 =</td>
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</tbody>
</table>
3. Think about this circle symbol:

Draw $3 \times$  

Draw $2 \times$  

Draw $1 \times$  

Draw $\frac{1}{2} \times$  

Draw $\frac{1}{3} \times$  

Draw $\frac{1}{4} \times$  

When you look at a multiplication problem, how can you predict when the product will be smaller than one of the factors?

4. Predict the product by circling a phrase:

a) $1.5 \times 90$  
   Product is larger than 90.  
   Product is less than 90.

b) $0.8 \times 74$  
   Product is greater than 74.  
   Product is less than 74.

c) $\frac{1}{3} \times 26$  
   Product is greater than 26.  
   Product is less than 26.

d) $2\frac{1}{4} \times 31$  
   Product is greater than 31.  
   Product is less than 31.

e) $\frac{3}{4} \times 5.9$  
   Product is greater than 5.9.  
   Product is less than 5.9.
f) 0.9 x 6½  
Product is greater than 6½.  
Product is less than 6½.

g) 4¾ x .007  
Product is greater than .007.  
Product is less than .007.

5. Fill in the table and look for patterns.

<table>
<thead>
<tr>
<th>Division</th>
<th>Quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 divided by 4 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 3 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 2 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 1 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by ½ =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 1/3 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by ¼ =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 1/6 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 0 ????</td>
<td>undefined</td>
</tr>
</tbody>
</table>

You divided 24 by numbers that kept getting smaller and smaller. What pattern do you see in the quotients?

When you divide a number is the quotient always smaller than that number? Explain.
6. Fill in the table and look for patterns.

<table>
<thead>
<tr>
<th>Division problem</th>
<th>Quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 divided by 4 =</td>
<td></td>
</tr>
<tr>
<td>60 divided by 3 =</td>
<td></td>
</tr>
<tr>
<td>60 divided by 2 =</td>
<td></td>
</tr>
<tr>
<td>60 divided by 1 =</td>
<td></td>
</tr>
<tr>
<td>60 divided by 0.5 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 0.4 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 0.3 =</td>
<td></td>
</tr>
<tr>
<td>24 divided by 0.2 =</td>
<td></td>
</tr>
<tr>
<td>60 divided by 0 ?????</td>
<td>undefined</td>
</tr>
</tbody>
</table>

Explain what happens to the product when you multiply by a number that is between 1 and 0.

1. Consider this diagram of 12 boxes:

   Draw a picture of the diagram divided by 4:
   (How many groups of 4 can be made?)
Draw a picture of the diagram divided by 3:
   (How many groups of 3 can be made?)

Draw a picture of the diagram divided by 2:
   (How many groups of 2 can be made?)

Draw a picture of the diagram divided by 1:
   (How many groups of 1 can be made?)

Draw a picture of the diagram divided by ½:
   (How many groups of ½ of a box can be made?)

Draw a picture of the diagram divided by 1/3:
   (How many groups of 1/3 of a box can be made?)

Draw a picture of the diagram divided by ¼:
   (How many groups of ¼ of a box can be made?)

What happens to the quotient as you divide by smaller and smaller numbers?
DIFFERENTIATION: MA.6.A.1.1

Students with disabilities

Adapt each activity by adjusting for complexity and time and number of repetitions. For instance, students with disabilities should spend a lot of time understanding halves and fourths and thirds before missing in fractions that have less commonly used denominators.

English Language Learners

Mathematical vocabulary and symbols can be confusing to ELL students. ELL students can benefit from developing their own mathematical dictionary. Terms such as quotient, product, factor, divisor, dividend, reciprocal, mixed number, numerator, denominator and decimal point might be included in the dictionary for this benchmark.

Illustrating a word like quotient with all of the symbolic ways to represent division can be very useful.

Extended Activities for Enrichment

Zeno was an interesting Greek philosopher who posed other famous dilemmas. Accelerated students can be encouraged to read more about Zeno’s paradox.

Other philosophers, like Aristotle, offered their own “solutions” to the paradox. Reading about those solutions involves interpretation of mathematical language that accelerated students view as a challenge. Students can intuitively discover the idea of limits, a basis of calculus.

Other students might a greater interest in investigating the many times that Zeno’s ideas turn up in literature and movies.
SAMPLE QUESTIONS FROM THE FCAT: MA.6.A.1.1

Sample Item 1 MC

Merrill baked a cake in the shape of a rectangular prism for a party. After the party, $\frac{1}{4}$ of the cake had not been eaten. Merrill froze $\frac{2}{3}$ of the remaining cake. A diagram of the portion eaten and the portion frozen is shown below.

Which expression can be used to find the fraction of the original whole cake Merrill froze?

★ A. $\frac{2}{3} \times \frac{1}{4}$
B. $\frac{2}{3} + \frac{1}{4}$
C. $\frac{2}{3} - \frac{1}{4}$
D. $\frac{2}{3} + \frac{1}{4}$

Item Context Mathematics
LINKS TO WEB-BASED INSTRUCTIONAL RESOURCES:  MA.6.A.1.1

1.  http://www.utdanacenter.org/tctm/downloads/TMT_Fall_05.pdf  This link is to an article from the Texas Mathematics Teacher. The article explains the mathematical reasoning behind the division of fractions algorithm.

2.  http://www.funbrain.com/cgi-bin/fract.cgi  This is a game that two students can compete in together. The answers must be simplified to be counted as correct.

3.  http://www.visualfractions.com/DivideCircle.html  This is a visual representation of dividing two fractions using circle parts. This might be useful for one-on-one tutoring for remediation.

4.  http://jwilson.coe.uga.edu/emt669/Student.Folders/Lewis.Millard/unit/DayOne.html  This website shows a few graphic examples of explaining multiplication of fractions. The ideas illustrated could be translated into similar problems that could be generated for students.

5.  http://whyslopes.com/etc/fractions/fractions12a_Division_of_%20Fractions.html  This illustrates a method of dividing fractions by thinking of each fraction as representing a distance.

6.  http://nlvm.usu.edu/en/nav/frames_asid_194_g_3_t_1.html?from=category_g_3_t_1.html  This is a recommended resource for illustrating multiplication of fractions using a grid model. Students can manipulate the size of the grids to model multiplication of two proper fractions.