



CBAL Mathematics: Assessment *of, for, and as* Learning

An Overview

March 2010

Topics

- Competency Model and Developmental Models
- Sample Summative Assessment design (for a year) and design of a particular Summative Assessment
- Tasks in a Summative Assessment
- Scoring of Summative Assessment tasks
- Brief Overview of Formative Assessment materials

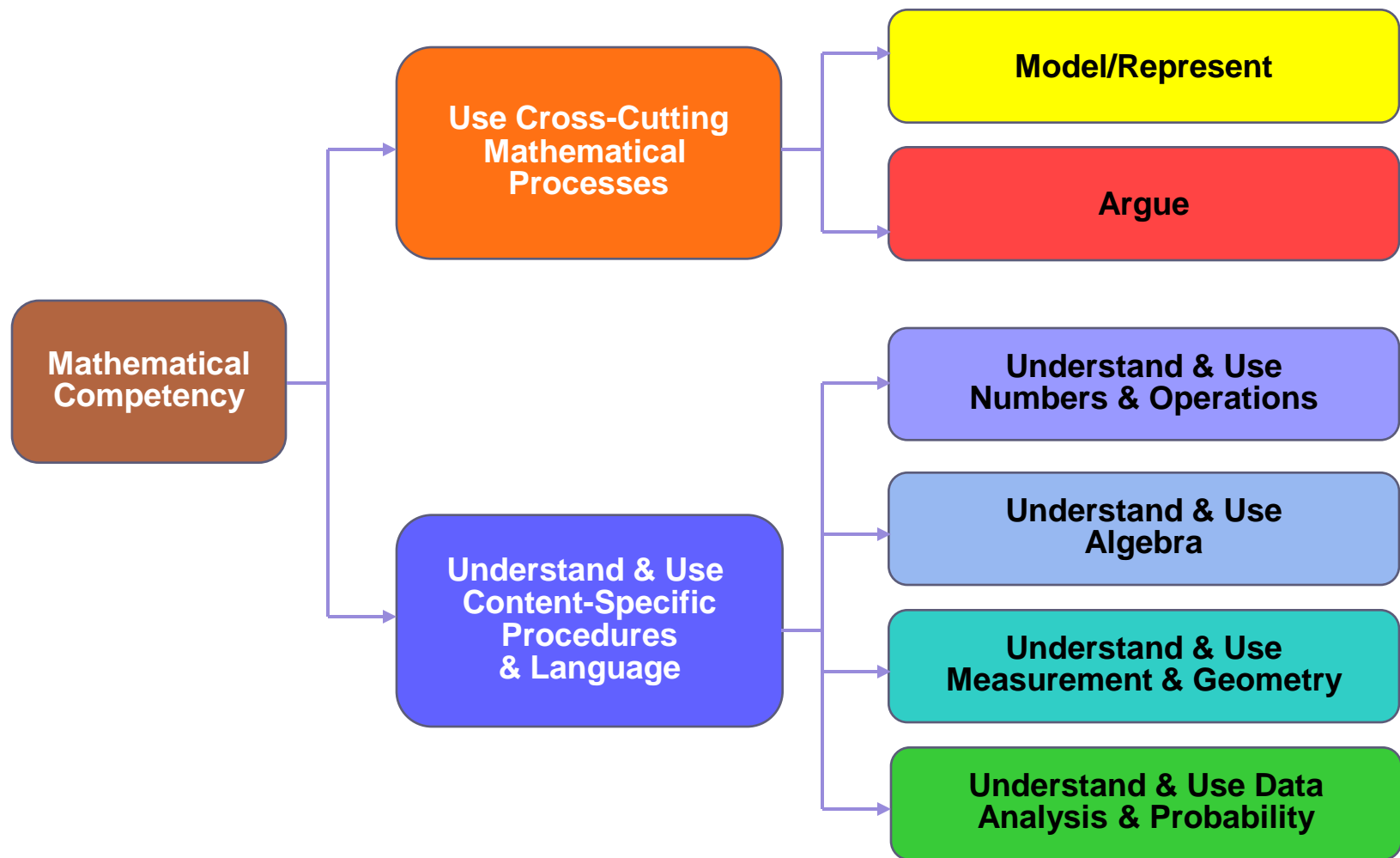
Overarching Considerations for Development of CBAL Mathematics Materials

- What does it mean to be mathematically competent?
- Which mathematical competencies will be measured and how can they best be measured (and therefore yield information that will help students improve their understanding of mathematics)?

CBAL Mathematics: Role of Competency Model

- Competency model expresses current understanding of the knowledge, skills, and abilities required by the discipline of mathematics, as informed by current cognitive and educational research.
- Levels of ability within the competency model reflect current understanding of the development of mathematical skills within the student population.
- Competency model is consistent with existing standards (e.g. NCTM, state-level frameworks, GAISE, Achieve benchmarks, and underlying research).

Middle School Math Competency Model: The First Three Levels



Middle School Mathematics



“Use Cross-Cutting Mathematical Processes” Branch of Competency Model

- There is an emphasis on developing instruments that assess the ability to reason, hypothesize, make conjectures, develop models, and construct justifications (all within the context of mathematics).
- These emphases are consistent with NCTM’s *Focus in High School Mathematics — Reasoning and Sense Making*.

This publication defines reasoning generally as “the process of drawing conclusions on the basis of evidence or stated assumptions.” It further states that “Although reasoning is an important part of all disciplines, it plays a special and fundamental role in mathematics.”

Sense making is defined as “developing understanding of a situation, context, or concept by connecting it with existing knowledge.”

Expanding Competency: Cognitive Developmental Models

- Cognitive developmental models define, from a cognitive perspective, the stages of competency through which students progress.
- They are explicit about changes (conceptual, procedural, strategic) that occur as a consequence of learning.
- They provide
 - A basis for defining a meaningful scale of measurement, and
 - A road map for supporting teaching and learning

Examples of Developmental Models

- Proportional Reasoning
- Algebra (equality, variable, function)
- Statistics (graphs, variation, central tendency, sampling/design)

Sample Summative Assessment Design for a Year

	Summative Assessment 1	Summative Assessment 2	Summative Assessment 3
16 questions; 45-60 minutes	Long task 1 (foundational)	Long task 2 (foundational)	Long task 3 (foundational)
7 questions; 20-30 minutes	Short task 1	Short task 3	Short task 5
7 questions; 20-30 minutes	Short task 2	Short task 4	Short task 6
Total Questions; Total time	30 questions; 90-120 min	30 questions; 90-120 min	30 questions; 90-120 min

Characteristics of Long (Foundational) Tasks

- Foundational tasks start with a big idea and are comprised of a series of questions based on the same task context that are scaffolded toward a culminating question. However, they are not just a set of questions set in a context.
- Foundational tasks always engage the student in sound mathematical practices. One of the two top-level cross-cutting process competencies (Model or Argue) will be the primary focus. However, the other competency will also be represented in the task.
- Foundational tasks are designed to be an indirect learning experience for students about mathematics as a discipline.

General Features of Summative Assessments

- Each of the two cross-cutting processes (Model and Argue) is used in at least two summative assessment administrations.
- Each of the four competencies in Understand and Use Content Specific Procedures and Language is used in at least two summative assessment administrations.
- All syllabus/standard topics (and as many subtopics as possible) are addressed at least once in the summative assessment administrations (and at least once in the formative assessments/activities).
- Long (foundational) tasks used for summative purposes typically require about 45-60 minutes to complete and tap a representative sampling of the constellation of concepts and skills that are associated with a particular part of the discipline of mathematics, focusing on critical developmental distinctions.

Example of Coverage of Competencies Across Three Summative Assessment Administrations

	Fall 2007	Spring 2008	Fall 2008
Cross-cutting Mathematical Processes	X	X	X
Model/Represent	X	X	X
Abstract to models and create representational devices	x	x	x
Revise and improve models and representational devices			x
Recognize improbable results			x
Recognize the effects of assumptions		x	
Apply models and interpret representational devices	x	x	x
Interrelate representational devices		x	x
Communicate	x	x	x
Argue/Justify	X	X	X
Develop a logical argument	x	x	x
Provide supporting examples			
Provide counterexamples			x
Pose conjectures	x	x	x
Evaluate arguments	x	x	x
Recognize need for formal proof			
Develop informal direct proof			
Understand and Use Content-Specific Procedures and Language	X	X	X
Understand and Use Numbers and Operations	X	X	X
Understand and Operate with Rational Numbers	x	x	x
Use Proportionality with Understanding	x	x	x
Reason about Quantities		x	x
Estimate and Approximate		x	x
Understand and Use Algebra	X	X	X
Solve equations and simplify expressions		x	
Use, graph, and reason about linear functions and some non-linear functions		x	x
Use and understand algebraic notation	x	x	x
Understand and Use Geometry and Measurement	X		X
Use formulas to determine perimeter and area of 2-dimensional shapes	x		
Use formulas to determine surface area and volume of 3-dimensional shapes			
Work in 2-and 3-dimensional Space Using Distance and Angle	x		
Understand Transformations and Properties of Similar Figures	x		x
Understand and Use Data Analysis and Probability		X	X
Calculate the theoretical probability of a single event			
Use and interpret descriptive statistics		x	x
Use and interpret data displays		x	x

Coverage of Competencies in One Summative Assessment Administration (Grade 8)

	Use Cross-cutting Mathematical Processes		Understand and Use Content-Specific Procedures and Language			
	Model/ Represent	Argue/ Justify	Understand & Use Numbers & Operations	Understand & Use Algebra	Understand & Use Measurement & Geometry	Understand & Use Data Analysis & Probability
PAA 1		LT	LT ST1		ST2	LT
PAA 2	LT	ST1	ST2	LT	ST1	
PAA 3	LT ST2	LT ST1	ST2	LT ST2	LT	LT ST1

LT: Long Task; ST1: Short Task 1; ST2: Short Task 2

Number of Questions In A Year of Summative Assessments (By Competency and Grade Level)

	Use Cross-cutting Mathematical Processes		Understand and Use Content-Specific Procedures and Language			
	Model/ Represent	Argue/ Justify	Understand & Use Numbers & Operations	Understand & Use Algebra	Understand & Use Measurement & Geometry	Understand & Use Data Analysis & Probability
Grade 8	22	21	31	21	20	7
Grade 7	22	6	30	18	24	0

Example of Tasks in a Summative Assessment Administration

Mathematical Focus: Ratios and Proportional Reasoning

- Short Task 1: Focuses on assessing an understanding of ratio in a variety of ways including part to part ratios, part to whole ratios, ratios as rates, and percents
- Short Task 2: Focuses on assessing an understanding of ratios involving rational numbers and linking ratio and proportional reasoning
- Long (Foundational) Task: Integrates assessment of proportional reasoning at different levels throughout task as the student works toward making a complex decision (with multiple conditions to evaluate) and justifying it

Example of Context for Short Task: Cartooning (Grade 7)

Question # Timer


Cartooning

CBAL MATH

Calc STOP Back Next

Cartooning

Michelle draws a cartoon called *Adam's Adventures* that appears in a daily newspaper. She uses some basic guidelines to make sure that the characters are drawn the same way each time.



Example of One Question in Cartooning Task

An online tool is available to students for use in this question.

Cartooning

Question #
2 of 9

Timer

CBAL^{MATH}

Calc

STOP

Back

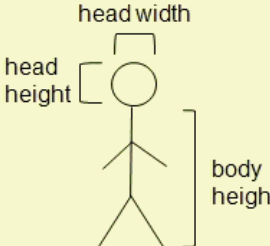
Next

Michelle has drawn the body for Adam, the main character in Adam's Adventure on the grid. When Michelle draws Adam,


- the ratio $\frac{\text{height of his head}}{\text{width of his head}} = \frac{4}{3}$
- the ratio $\frac{\text{height of his head}}{\text{height of his body}} = \frac{2}{5}$

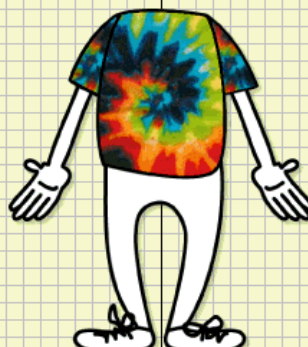
head width

head height



body height

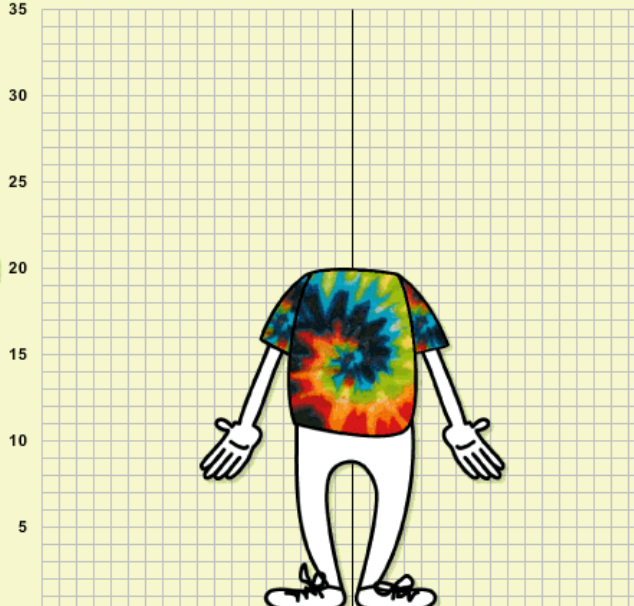




On the right, use your cursor to drag Adam's head to his body and make it the correct size and shape on the grid. You can resize the head by pulling the handles that appear near the head when you select it.

$$\frac{\text{head height}}{\text{head width}} = \frac{5}{5} = \frac{3}{3}$$

$$\frac{\text{head height}}{\text{body height}} = \frac{5}{20} = \frac{1.3}{5}$$



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Example of Context for Long Task: Mix It Up (Grade 7)

Mix It Up Question # Timer


CBAL MATH

Calc STOP Back Next

Mix It Up

Your class is helping fix up a teen center.

The class has received a donation of red paint and blue paint and has decided to mix them to get purple paint to paint the teen center.



The illustration shows a two-story building with a brown facade and blue trim. A sign on the roof reads "Teen Center" in red, slanted letters. There is a blue awning over the entrance, and a green bush is in the foreground on the left.

Mix It Up: Online Simulation Tool

Mix It Up Question # 1 of 10 Timer

CBAL MATH

Mix Calc STOP Back Next

Paint Mixing Tool

Click on the **Add Red** and **Add Blue** buttons to experiment with mixing paint so that you can see how the ratio of red to blue affects the color of the paint.

Click on **Start Over** to empty the can and begin a new experiment.

Add Red

Add Blue

Red: 0

Blue: 0

Start Over



Culminating Question in Mix It Up (Grade 7)

Mix It Up

Question #

10 of 10

Timer

CBAL

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Mix

Calc

STOP

Back

Next

Refer to the tables below when answering this question.

Ratios of Shades of Purple

	Red Paint : Blue Paint	$\frac{\text{Red Paint}}{\text{Blue Paint}}$
Rosie	2:3	$\frac{2}{3}$
Karen	5:6	$\frac{5}{6}$

Maximum Amount of Each Shade Using Donated Paint
(18 quarts of red and 24 quarts of blue were donated)

	Donated Red Used	Donated Blue Used	Total Amount of Paint Made	Donated Red Left	Donated Blue Left
Rosie	16	24	40	2	0
Karen	15	18	33	3	6

Keep these conditions in mind when answering the question below.

- All of the donated paint should be used.
- To save money, the least amount of additional paint will be purchased so that all the donated paint can be used.
- The class will paint the teen center in either Rosie's or Karen's shade.
- When red and blue are mixed together, only full quarts of paint are used.

Which ratio of red paint to blue paint do you recommend be chosen to meet all of the conditions above?

Rosie's
 Karen's

Explain your answer.

Another Example of an Online Tool

Formative assessment tasks also include tools to support the mathematics covered in the summative assessments and to help students increase their understanding of certain concepts; in this instance, ratio.

The screenshot shows a web-based interface for a 'Sweetness Meter'. The title is 'Sweetness Meter' with the subtitle 'Scoops of dissolved mix : Cups of water'. On the left is a vertical thermometer-like scale with a red bulb at the bottom. The scale has tick marks and labels for various ratios: 2:1, 11:6, 9:5, 7:4, 5:3, 8:5, 3:2, 7:5, 4:3, 5:4, 6:5, 7:6, 1:1, 5:6, 4:5, 3:4, 2:3, 3:5, 1:2, 2:5, 1:3, 1:4, 1:5, 1:6, and 0. To the right of the scale are two input fields: 'Scoops of mix : 0' with an 'Add Mix' button, and 'Cups of water : 0' with an 'Add Water' button. Below these is a text instruction: 'Assume that adding scoops of punch mix does not add volume to the water' and a 'Start Over' button. On the right side of the interface, there is a 3D illustration of a glass pitcher on a wooden surface. Above the pitcher are two small metal scoops: one containing red liquid and the other containing blue liquid.

Summative Assessments as Models of Good Instruction and Learning Practice

- Focus on significant mathematical concepts and content
- Connect acquired mathematical knowledge to new situations
- Support correct mathematical thinking and disposition
- Are structured so that questions within a task are designed to link to cognitive developmental models, thereby potentially enhancing information provided by competency model and better informing instructional decisions

Scoring the Summative Assessment Tasks

- **Automated scoring:** To date, one-half to two-thirds of the responses have been scored using automated methods. The remaining responses have been scored by humans.
- A new interface is planned for 2010 that will increase the amount of automated scoring in the assessment.

The Formative Materials

- Supports effective formative assessment by providing a library of classroom tasks that are linked to competency model to elicit evidence of student understanding and promote student learning
- Contains two types of tasks:
 - Longer classroom tasks (fulfill all characteristics of foundational tasks and integrate content strands)
 - Shorter focused tasks (develop major mathematical concepts or concepts that are instructionally difficult)
- Contains supporting materials:
 - features of a model response
 - likely student responses and mapping to developmental model
 - suggested pedagogical content strategies
 - options for further instruction
 - suggested discussion questions
 - possible misconceptions/misunderstandings

Formative Assessment Materials

(Selected Differences between Formative and Summative Assessments)

- Some of the questions in the formative assessment tasks are designed to be more 'open' in nature than those in the summative assessment tasks. This helps to encourage teacher-student discussion and provides a way for the teacher to delve more deeply into the extent to which the student understands the material.
- Some tasks in the formative assessments include activities such as *extended* explorations with online simulation tools and formal small group assignments that help to promote deeper understanding of core mathematical concepts and practices.

Summary

- The competency model broadly defines the domain of mathematics for middle school students based on the results of cognitive science research. Research supports the assessment of relatively sophisticated mathematical practices such as modeling and constructing arguments at an early age; the ability to effectively assess these practices through the use of summative assessment tasks is a target of CBAL.
- The cognitive developmental models represent how students typically learn the key concepts and skills.
- Joining both the competency and appropriate developmental models in the creation of formative and summative tasks may facilitate the assessment of students' understanding in key concepts and skills, and may help to inform instruction that moves students to the next developmental level.