

MATHEMATICS TEST
SAMPLE QUESTIONS

The following questions illustrate the range of the test in terms of abilities measured, the disciplines covered, and the difficulty of the questions posed. They should not, however, be considered representative of the entire scope of the test in either content or difficulty. An answer key follows the questions.

1. A student is given an exam consisting of 8 essay questions divided into 4 groups of 2 questions each. The student is required to select a set of 6 questions to answer, including at least 1 question from each of the 4 groups. How many sets of questions satisfy this requirement?

(A) 6
(B) 24
(C) 28
(D) 48
(E) 96
2. The function f is differentiable on the interval $(0, 4)$. If $f(1) = 1$ and $f(3) = 7$, then there is at least one c in $(1, 3)$ such that $f'(c) =$

(A) -1
(B) 0
(C) 1
(D) 2
(E) 3
3. Let A and B be metric spaces, and let $f : A \rightarrow B$. Suppose that whenever X is an open set in B , the set $\{a \in A : f(a) \notin X\}$ is closed in A . Which of the following must be true?

I. f is injective.
II. f is continuous.
III. f is a homeomorphism.

(A) None
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III
4. In the xy -plane, the line tangent to the graph of $x^2 + xy + y^2 = 3$ at the point $(1, 1)$ has a slope of

(A) -3
(B) -1
(C) 0
(D) $\frac{1}{3}$
(E) 1
5. Let \mathbb{Z} be the ring of integers, and let R be a ring without identity. Let $S = \mathbb{Z} \times R$ be the ring with addition and multiplication defined by $(k, a) + (n, b) = (k + n, a + b)$ and $(k, a)(n, b) = (kn, kb + na + ab)$, where k and n are in \mathbb{Z} , and a and b are in R . Which of the following must be true about S ?

I. S is a ring with identity.
II. S has a subring isomorphic to R .
III. S is an integral domain (it has no zero-divisors).

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

$$\frac{dQ}{dt} = 6(5 - Q(t))$$

$$Q(0) = 0$$

6. The function $Q(t)$ satisfies the differential equation shown above. What is the value of t such that $Q(t) = 4$?

- (A) $\frac{13}{3}$
 (B) $\frac{\ln 5}{6}$
 (C) $\frac{\ln 6}{5}$
 (D) $30 - \frac{\ln 5}{6}$
 (E) $30 + \frac{\ln 6}{5}$

7. What are the eigenvalues of $\begin{pmatrix} 6 & -3 \\ 1 & 2 \end{pmatrix}$?

- (A) 1 and 15
 (B) 2 and 6
 (C) 3 and 5
 (D) $\frac{5}{2} + \frac{i\sqrt{15}}{2}$ and $\frac{5}{2} - \frac{i\sqrt{15}}{2}$
 (E) $4 + i$ and $4 - i$

8. What is the area of the portion of the surface $z = x^2 + y^2$ lying inside the cylinder $x^2 + y^2 = 4$ in xyz -space?

- (A) 21π
 (B) $\frac{21\pi}{2}$
 (C) $\frac{\pi}{3} \left(17^{\frac{3}{2}} \right)$
 (D) $\frac{\pi}{2} \left(17^{\frac{3}{2}} - 1 \right)$
 (E) $\frac{\pi}{6} \left(17^{\frac{3}{2}} - 1 \right)$

9. $\int_{-1}^1 \frac{e^x - e^{-x}}{e^{2x} + e^{-2x} + 2} dx =$

- (A) 0
 (B) 1
 (C) $2e$
 (D) $e + e^{-1}$
 (E) $e^2 - e^{-2}$

10. If V_n is the real vector space of all n -tuples of real numbers for each $n > 1$, which of the following must be true?

- I. Every basis of V_n contains exactly n vectors.
 II. Every basis of V_n is an orthogonal set of vectors.
 III. Every set of $n + 1$ vectors of V_n is a linearly dependent set.

- (A) I only
 (B) II only
 (C) I and II
 (D) I and III
 (E) II and III

ANSWER KEY

1. B
 2. E
 3. B
 4. B
 5. C
 6. B
 7. C
 8. E
 9. A
 10. D