Quantitative Reasoning Practice Questions

This document contains four groups of GRE® Quantitative Reasoning practice questions. Each of the first three practice groups consists of Quantitative Comparison questions, both types of Multiple-choice questions, and Numeric-Entry questions. These three groups are arranged in order of increasing difficulty. The first is easy, the second is medium, and the third is hard. The fourth practice group consists of Data Interpretation questions of varying levels of difficulty.

Following the last group is an Answer Key for quick reference.

In a separate document, titled Quantitative Reasoning Practice Questions with Explanations, are complete solutions for every question. Each explanation is presented with the corresponding question, so that you can easily see what was asked and what the various answer choices or Numeric-Entry answer boxes were.

Sharpen your GRE Quantitative Reasoning skills by working your way through these groups of questions. For the first three groups of questions, begin with the easy group and then move on to the medium and hard groups. Review the answers and explanations carefully, paying particular attention to explanations for questions that you answered incorrectly.

For the practice questions in this document, use the directions that begin on the following page.
General Directions

For each question, indicate the best answer, using the directions given.

Notes: All numbers used are real numbers.

All figures are assumed to lie in a plane unless otherwise indicated.

Geometric figures, such as lines, circles, triangles, and quadrilaterals, are not necessarily drawn to scale. That is, you should not assume that quantities such as lengths and angle measures are as they appear in a figure. You should assume, however, that lines shown as straight are actually straight, points on a line are in the order shown, and more generally, all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities from how they are drawn in the geometric figure.

Coordinate systems, such as xy-planes and number lines, are drawn to scale; therefore, you can read, estimate, or compare quantities in such figures from how they are drawn in the coordinate system.

Graphical data presentations, such as bar graphs, circle graphs, and line graphs, are drawn to scale; therefore, you can read, estimate, or compare data values from how they are drawn in the graphical data presentation.
Directions for Quantitative Comparison Questions

Compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices.

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.

A symbol that appears more than once in a question has the same meaning throughout the question.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
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Example 1: \((2)(6)\) \(2 + 6\)
The correct answer choice for Example 1 is (A). \((2)(6)\), or 12, is greater than \(2 + 6\), or 8.

Example 2: \(PS\) \(SR\)
The correct answer choice is (D). The relationship between \(PS\) and \(SR\) cannot be determined from the information given since equal measures cannot be assumed, even though \(PS\) and \(SR\) appear to be equal in the figure.
Directions for Numeric-Entry Questions

These questions require a number to be entered by circling entries in a grid. If you are not entering your own answers, your scribe should be familiar with these instructions.

1. Your answer may be an integer, a decimal, or a fraction, and it may be negative.
2. Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Although fractions do not need to be reduced to lowest terms, they may need to be reduced to fit in the grid.
3. Enter the exact answer unless the question asks you to round your answer.
4. If a question asks for a fraction, the grid will have a built-in division slash (/). Otherwise, the grid will have a decimal point.
5. Start your answer in any column, space permitting. Circle no more than one entry in any column of the grid. Columns not needed should be left blank.
6. Write your answer in the boxes at the top of the grid and circle the corresponding entries. **You will receive credit only if your grid entries are clearly marked, regardless of the number written in the boxes at the top.**
Examples of acceptable ways to use the grid:
Integer answer:  502 (either position is correct)
Decimal answer: $-4.13$

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Fraction answer: $-\frac{2}{10}$

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Group 1. Discrete Questions: Easy

Questions 1–6: Quantitative Comparison Questions.

Directions for answering these questions can be found on page 4 of this document.

Emma spent $75 buying a used bicycle and $27 repairing it. Then she sold the bicycle for 40 percent more than the total amount she spent buying and repairing it.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The price at which Emma sold the bicycle</td>
<td>$140</td>
</tr>
</tbody>
</table>

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.
In the figure above, squares $PQRV$ and $VRST$ have sides of length 6.

**Quantity A**

2. The area of the shaded region

- **A** Quantity A is greater.
- **B** Quantity B is greater.
- **C** The two quantities are equal.
- **D** The relationship cannot be determined from the information given.
In 2009 the property tax on each home in Town X was \( p \) percent of the assessed value of the home, where \( p \) is a constant. The property tax in 2009 on a home in Town X that had an assessed value of $125,000 was $2,500.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
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<tbody>
<tr>
<td>3. The property tax in 2009 on a home in Town X that had an assessed value of $160,000</td>
<td>$3,000</td>
</tr>
</tbody>
</table>

- A Quantity A is greater.
- B Quantity B is greater.
- C The two quantities are equal.
- D The relationship cannot be determined from the information given.
\[ x + y = -1 \]

**Quantity A** | **Quantity B**
--- | ---
4. \( x \) | \( y \)

\( A \) Quantity A is greater.
\( B \) Quantity B is greater.
\( C \) The two quantities are equal.
\( D \) The relationship cannot be determined from the information given.

\( r, s, \) and \( t \) are three consecutive odd integers such that \( r < s < t \).

**Quantity A** | **Quantity B**
--- | ---
5. \( r + s + 1 \) | \( s + t - 1 \)

\( A \) Quantity A is greater.
\( B \) Quantity B is greater.
\( C \) The two quantities are equal.
\( D \) The relationship cannot be determined from the information given.
6. The slope of line $k$  The slope of line $m$

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.
Questions 7–11: Multiple-choice Questions—Select One Answer Choice.

Each of these questions has five answer choices. Select the best one of the answer choices given.

7. In the figure above, what is the value of \( \frac{x + y + z}{45} \) ?

A 2  
B 3  
C 4  
D 5  
E 6
8. A certain store sells two types of pens: one type for $2 per pen and the other type for $3 per pen. If a customer can spend up to $25 to buy pens at the store and there is no sales tax, what is the greatest number of pens the customer can buy?

A  9
B  10
C  11
D  12
E  20

9. If \( y = 3x \) and \( z = 2y \), what is \( x + y + z \) in terms of \( x \) ?

A 10x
B 9x
C 8x
D 6x
E 5x
10. A certain shipping service charges an insurance fee of $0.75 when shipping any package with contents worth $25.00 or less and an insurance fee of $1.00 when shipping any package with contents worth over $25.00. If Dan uses the shipping company to ship three packages with contents worth $18.25, $25.00, and $127.50, respectively, what is the total insurance fee that the company charges Dan to ship the three packages?

A $1.75  
B $2.25  
C $2.50  
D $2.75  
E $3.00
11. If 55 percent of the people who purchase a certain product are female, what is the ratio of the number of females who purchase the product to the number of males who purchase the product?

A  11 to 9
B  10 to 9
C  9 to 10
D  9 to 11
E  5 to 9
Questions 12 and 13: Numeric-Entry Questions

Directions for answering Numeric-Entry questions can be found on page 5 of this document.
To answer question 12, enter a number in the answer space provided. The number can include a decimal point, and can be positive, negative, or zero. The number entered cannot be a fraction.

12. In the rectangular solid above, the length of $TU$ is 3, the length of $UV$ is 4, and the length of $VR$ is 2. What is the area of the shaded rectangular region?
To answer question 13, enter a number in the answer space provided. The number can include a decimal point, and can be positive, negative, or zero. The number entered cannot be a fraction.

13. A list of numbers has a mean of 8 and a standard deviation of 2.5. If \( x \) is a number in the list that is 2 standard deviations above the mean, what is the value of \( x \)?

\[
x = 
\begin{array}{cccccccc}
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\
5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \\
\end{array}
\]
Question 14: Multiple-choice Question — Select One or More Answer Choices.

Question 14 has five answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, or as many as all five of the answer choices.

The circle graph above shows the distribution of 200,000 physicians by specialty. Which of the following sectors of the circle graph represent more than 40,000 physicians? Indicate all such sectors.

A  Pediatrics
B  Internal Medicine
C  Surgery
D  Anesthesiology
E  Psychiatry

14. The circle graph above shows the distribution of 200,000 physicians by specialty. Which of the following sectors of the circle graph represent more than 40,000 physicians?

Indicate all such sectors.
Group 2. Discrete Questions: Medium

Questions 1–5: Quantitative Comparison Questions

Directions for answering these questions can be found on page 4 of this document.

Machine $R$, working alone at a constant rate, produces $x$ units of a product in 30 minutes, and machine $S$, working alone at a constant rate, produces $x$ units of the product in 48 minutes, where $x$ is a positive integer.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The number of units of the product that machine $R$, working alone at its constant rate, produces in 3 hours</td>
<td>The number of units of the product that machine $S$, working alone at its constant rate, produces in 4 hours</td>
</tr>
</tbody>
</table>

Option A: Quantity A is greater.
Option B: Quantity B is greater.
Option C: The two quantities are equal.
Option D: The relationship cannot be determined from the information given.
List X and list Y each contain 60 numbers. Frequency distributions for each list are given above. The average (arithmetic mean) of the numbers in list X is 2.7, and the average of the numbers in list Y is 7.1. List Z contains 120 numbers: the 60 numbers in list X and the 60 numbers in list Y.

Quantity A: The average of the 120 numbers in list Z
Quantity B: The median of the 120 numbers in list Z

A  Quantity A is greater.
B  Quantity B is greater.
C  The two quantities are equal.
D  The relationship cannot be determined from the information given.
In the figure above, the diameter of the circle is 10.

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<th>Quantity A</th>
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<tr>
<td>3. The area of quadrilateral $ABCD$</td>
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</table>

A  Quantity A is greater.
B  Quantity B is greater.
C  The two quantities are equal.
D  The relationship cannot be determined from the information given.
\[ x^2y > 0 \]
\[ xy^2 < 0 \]

Quantity A

\[ x \]

Quantity B

\[ y \]

\[ \begin{array}{r} \text{A} \quad \text{Quantity A is greater.} \\
\text{B} \quad \text{Quantity B is greater.} \\
\text{C} \quad \text{The two quantities are equal.} \\
\text{D} \quad \text{The relationship cannot be determined from the information given.} \end{array} \]
Among the 9,000 people attending a football game at College C, there were $x$ students from College C and $y$ students who were not from College C.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
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<tbody>
<tr>
<td>The number of people</td>
<td>$9,000 - x - y$</td>
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<tr>
<td>attending the game who were not students</td>
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</table>

A  Quantity A is greater.
B  Quantity B is greater.
C  The two quantities are equal.
D  The relationship cannot be determined from the information given.
Questions 6–10: Multiple-choice Questions — Select One Answer Choice.

Each of these questions has five answer choices. Select the best one of the answer choices given.

6. If \( x \neq 0 \), which of the following is equivalent to \( \frac{x(x^2)^3}{x^2} \) ?

- **A** \( x^2 \)
- **B** \( x^3 \)
- **C** \( x^4 \)
- **D** \( x^5 \)
- **E** \( x^6 \)
7. The figure above shows the graph of the function $f$ in the $xy$-plane. What is the value of $f(f(-1))$?

A. $-2$
B. $-1$
C. 0
D. 1
E. 2
8. If \( \frac{d - 3n}{7n - d} = 1 \), which of the following statements describes \( d \) in terms of \( n \)?

A) \( d \) is 4 less than \( n \).

B) \( d \) is 4 more than \( n \).

C) \( d \) is \( \frac{3}{7} \) of \( n \).

D) \( d \) is 2 times \( n \).

E) \( d \) is 5 times \( n \).
9. By weight, liquid A makes up 8 percent of solution R and 18 percent of solution S. If 3 grams of solution R are mixed with 7 grams of solution S, then liquid A accounts for what percent of the weight of the resulting solution?

A  10%
B  13%
C  15%
D  19%
E  26%
10. Of the 700 members of a certain organization, 120 are lawyers. Two members of the organization will be selected at random. Which of the following is closest to the probability that neither of the members selected will be a lawyer?

A  0.5
B  0.6
C  0.7
D  0.8
E  0.9
Questions 11 and 12: Numeric-Entry Questions

Directions for answering Numeric-Entry questions can be found on page 5 of this document.

To answer question 11, enter a number in the answer space provided. The number can include a decimal point, and can be positive, negative, or zero. The number entered cannot be a fraction.
11. The figure above represents a rectangular garden with a walkway around it. The garden is 18 feet long and 12 feet wide. The walkway is uniformly 3 feet wide, and its edges meet at right angles. What is the area of the walkway?
To answer question 12 enter a fraction in the answer space provided. The fraction can be positive or negative. Neither the numerator nor the denominator of the fraction can include a decimal point. The fraction does not have to be in lowest terms.

12. Line $k$ lies in the $xy$-plane. The $x$-intercept of line $k$ is $-4$, and line $k$ passes through the midpoint of the line segment whose endpoints are $(2, 9)$ and $(2, 0)$. What is the slope of line $k$?

Give your answer as a fraction.
Questions 13 and 14: Multiple-choice Questions—Select One or More Answer Choices.

Question 13 has four answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, or as many as all four of the answer choices.

13. If the lengths of two sides of a triangle are 5 and 9, respectively, which of the following could be the length of the third side of the triangle?

   Indicate all such lengths.

   A  3
   B  5
   C  8
   D  15
Question 14 has three answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, or as many as all three of the answer choices.

14. On the number line shown above, the tick marks are equally spaced. Which of the following statements about the numbers \(x\), \(y\), and \(z\) must be true?

Indicate all such statements.

A. \(xyz < 0\)

B. \(x + z = y\)

C. \(z(y - x) > 0\)
In the figure above, $ABCD$ is a parallelogram.

Quantity A | Quantity B
---|---
1. The area of $ABCD$ | 24

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.
In the course of an experiment, 95 measurements were recorded, and all of the measurements were integers. The 95 measurements were then grouped into 7 measurement intervals. The graph above shows the frequency distribution of the 95 measurements by measurement interval.

**Quantity A**

The average (arithmetic mean) of the 95 measurements

**Quantity B**

The median of the 95 measurements

A   Quantity A is greater.
B   Quantity B is greater.
C   The two quantities are equal.
D   The relationship cannot be determined from the information given.
$x$ is an integer greater than 1.

3.  

<table>
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<tr>
<th>Quantity A</th>
<th>Quantity B</th>
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<tbody>
<tr>
<td>$3^x + 1$</td>
<td>$4^x$</td>
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</table>

- A  Quantity A is greater.
- B  Quantity B is greater.
- C  The two quantities are equal.
- D  The relationship cannot be determined from the information given.

A, B, and C are three rectangles. The length and width of rectangle A are 10 percent greater and 10 percent less, respectively, than the length and width of rectangle C. The length and width of rectangle B are 20 percent greater and 20 percent less, respectively, than the length and width of rectangle C.

4.  

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>The area of rectangle A</td>
<td>The area of rectangle B</td>
</tr>
</tbody>
</table>

- A  Quantity A is greater.
- B  Quantity B is greater.
- C  The two quantities are equal.
- D  The relationship cannot be determined from the information given.
The random variable \( X \) is normally distributed. The values 650 and 850 are at the 60th and 90th percentiles of the distribution of \( X \), respectively.

Quantity A | Quantity B
---|---
5. The value at the 75th percentile of the distribution of \( X \) | 750

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.
Set $S$ consists of all positive integers less than 81 that are not equal to the square of an integer.

**Quantity A**

6. The number of integers in set $S$

**Quantity B**

72

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.
Questions 7–12: Multiple-choice Questions—Select One Answer Choice.

Each of these questions has five answer choices. Select the best one of the answer choices given.

7. A manager is forming a 6-person team to work on a certain project. From the 11 candidates available for the team, the manager has already chosen 3 to be on the team. In selecting the other 3 team members, how many different combinations of 3 of the remaining candidates does the manager have to choose from?

A  6
B  24
C  56
D  120
E  462
8. Which of the following could be the graph of all values of \( x \) that satisfy the inequality \( 2 - 5x \leq \frac{6x - 5}{3} \)?
9. If \( 1 + x + x^2 + x^3 = 60 \), then the average (arithmetic mean) of \( x \), \( x^2 \), \( x^3 \), and \( x^4 \) is equal to which of the following?

- \( A \) 12x
- \( B \) 15x
- \( C \) 20x
- \( D \) 30x
- \( E \) 60x
10. Parallelogram $OPTS$ lies in the $xy$-plane, as shown in the figure above. The coordinates of point $P$ are $(2, 4)$ and the coordinates of point $T$ are $(8, 6)$. What are the coordinates of point $S$?

A (3, 2)  
B (3, 3)  
C (4, 4)  
D (5, 2)  
E (6, 2)
11. The relationship between the area $A$ of a circle and its circumference $C$ is given by the formula $A = kC^2$, where $k$ is a constant. What is the value of $k$?

A) $\frac{1}{4\pi}$

B) $\frac{1}{2\pi}$

C) $\frac{1}{4}$

D) $2\pi$

E) $4\pi^2$
12. The sequence of numbers \( a_1, a_2, a_3, \ldots, a_n, \ldots \) is defined by 
\[
a_n = \frac{1}{n} - \frac{1}{n + 2}
\]
for each integer \( n \geq 1 \). What is the sum of the first 20 terms of this sequence?

\[\text{A} \quad \left(1 + \frac{1}{2}\right) - \frac{1}{20}\]

\[\text{B} \quad \left(1 + \frac{1}{2}\right) - \left(\frac{1}{21} + \frac{1}{22}\right)\]

\[\text{C} \quad 1 - \left(\frac{1}{20} + \frac{1}{22}\right)\]

\[\text{D} \quad 1 - \frac{1}{22}\]

\[\text{E} \quad \frac{1}{20} - \frac{1}{22}\]
Question 13: Numeric-Entry Question

Directions for answering Numeric-Entry questions can be found on page 5 of this document.

To answer question 13, enter a number in the answer space provided. The number can include a decimal point, and can be positive, negative, or zero. The number entered cannot be a fraction.

<table>
<thead>
<tr>
<th>$Y$</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>2</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>7</td>
</tr>
<tr>
<td>$\frac{5}{4}$</td>
<td>8</td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>8</td>
</tr>
<tr>
<td>$\frac{7}{4}$</td>
<td>9</td>
</tr>
</tbody>
</table>

13. The table above shows the frequency distribution of the values of a variable $Y$. What is the mean of the distribution? Give your answer to the nearest 0.01.
Questions 14 and 15: Multiple-choice Questions — Select One or More Answer Choices.

To answer these questions, select all the answer choices that apply.

Question 14 has four answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, or as many as all four of the answer choices.

14. Let \( S \) be the set of all positive integers \( n \) such that \( n^2 \) is a multiple of both 24 and 108. Which of the following integers are divisors of every integer \( n \) in \( S \)?

    Indicate all such integers.

    A   12
    B   24
    C   36
    D   72
15. The range of the heights of the female students in a certain class is 13.2 inches, and the range of the heights of the male students in the class is 15.4 inches.

Which of the following statements individually provide(s) sufficient additional information to determine the range of the heights of all the students in the class?

Indicate all such statements.

A  The tallest male student in the class is 5.8 inches taller than the tallest female student in the class.
B  The median height of the male students in the class is 1.1 inches greater than the median height of the female students in the class.
C  The average (arithmetic mean) height of the male students in the class is 4.6 inches greater than the average height of the female students in the class.
Group 4. Data Interpretation Sets

For Questions 1–7, select a single answer choice unless otherwise directed.

Questions 1–3 are based on the following data.

Percent of Female Faculty and Percent of Male Faculty at University X, by Field

- Total female faculty: 200
- Total male faculty: 250

- Biological Sciences
- Business
- Education
- Engineering
- Fine Arts
- Health Sciences
- Humanities
- Physical Sciences
- Social Sciences
- Other

Percent
Question 1 is a medium difficulty question.

1. There are 275 students in the field of engineering at University X. Approximately what is the ratio of the number of students in engineering to the number of faculty in engineering?

   A  8 to 1  
   B  10 to 1  
   C  12 to 1  
   D  14 to 1  
   E  20 to 1
Question 2 is a medium difficulty question.

2. Approximately what percent of the faculty in humanities are male?

A  35%
B  38%
C  41%
D  45%
E  51%
Question 3 is a hard question.

Question 3 is a Numeric-Entry question. To answer the question enter a fraction in the answer space provided. The fraction can be positive or negative. Neither the numerator nor the denominator of the fraction can include a decimal point. The fraction does not have to be in lowest terms.

3. For the biological sciences and health sciences faculty combined, \( \frac{1}{3} \) of the female and \( \frac{2}{9} \) of the male faculty members are tenured professors. What fraction of all the faculty members in those two fields combined are tenured professors?

\[
\begin{array}{cccc}
\text{ } & \text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } & \text{ } \\
- & 0 & 0 & 0 \\
0 & 1 & 1 & 1 \\
1 & 2 & 2 & 2 \\
2 & 3 & 3 & 3 \\
3 & 4 & 4 & 4 \\
4 & 5 & 5 & 5 \\
5 & 6 & 6 & 6 \\
6 & 7 & 7 & 7 \\
7 & 8 & 8 & 8 \\
8 & 9 & 9 & 9 \\
9 & 0 & 0 & 0 \\
\end{array}
\]
Questions 4–7 are based on the following data.

Value of Imports to and Exports from Country T, 2000–2009
(in United States dollars)

*1 billion = 1,000,000,000*
Question 4 is an easy question.

Question 4 has eight answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, or as many as all eight of the answer choices.

4. For which of the eight years from 2001 to 2008 did exports exceed imports by more than $5 billion? Indicate all such years.

A 2001
B 2002
C 2003
D 2004
E 2005
F 2006
G 2007
H 2008
Question 5 is a medium difficulty question.

5. Which of the following is closest to the average (arithmetic mean) of the 9 changes in the value of imports between consecutive years from 2000 to 2009?
   - A $260 million
   - B $320 million
   - C $400 million
   - D $480 million
   - E $640 million

Question 6 is a medium difficulty question.

6. In 2008 the value of exports was approximately what percent greater than the value of imports?
   - A 40%
   - B 60%
   - C 70%
   - D 120%
   - E 140%
Question 7 is a hard question.

7. If it were discovered that the value of imports shown for 2007 was incorrect and should have been $5 billion instead, then the average (arithmetic mean) value of imports per year for the 10 years shown would have been approximately how much less?

- A  $200 million
- B  $50 million
- C  $20 million
- D  $7 million
- E  $5 million
ANSWER KEYS

Group 1. Discrete Questions: Easy

1. **Choice A:** Quantity A is greater.

2. **Choice C:** The two quantities are equal.

3. **Choice A:** Quantity A is greater.

4. **Choice D:** The relationship cannot be determined from the information given.

5. **Choice B:** Quantity B is greater.

6. **Choice A:** Quantity A is greater.

7. **Choice C:** 4

8. **Choice D:** 12

9. **Choice A:** 10x

10. **Choice C:** $2.50

11. **Choice A:** 11 to 9
12. Question 12 is a numeric entry question. In question 12 you were asked to enter either an integer or a decimal. The answer to question 12 is 10.

13. Question 13 is a numeric entry question. In question 13 you were asked to enter either an integer or a decimal. The answer to question 13 is 13.

14. The correct answer to question 14 consists of 3 answer choices.
   
   Choice A: Pediatrics
   AND
   Choice B: Internal Medicine
   AND
   Choice C: Surgery

Group 2. Discrete Questions: Medium

1. Choice A: Quantity A is greater.

2. Choice B: Quantity B is greater.

3. Choice D: The relationship cannot be determined from the information given.

4. Choice B: Quantity B is greater.

5. Choice C: The two quantities are equal.
6. Choice D: \( x^5 \)

7. Choice D: 1

8. Choice E: \( d \) is 5 times \( n \).

9. Choice C: 15%

10. Choice C: 0.7

11. Question 11 is a numeric entry question. In question 11 you were asked to enter either an integer or a decimal. The answer to question 11 is 216.

12. Question 12 is a numeric entry question. In question 12 you were asked to enter a fraction. The answer to question 12 is \( \frac{3}{4} \) (or any equivalent fraction).

13. The correct answer to question 13 consists of 2 answer choices.  
    Choice B: 5
    AND  
    Choice C: 8
14. The correct answer to question 14 consists of 3 answer choices.
   - **Choice A:** $xyz < 0$
   - **AND**
   - **Choice B:** $x + z = y$
   - **AND**
   - **Choice C:** $z(y - x) > 0$

**Group 3. Discrete Questions: Hard**

1. **Choice B:** Quantity B is greater.

2. **Choice A:** Quantity A is greater.

3. **Choice D:** The relationship cannot be determined from the information given.

4. **Choice A:** Quantity A is greater.

5. **Choice B:** Quantity B is greater.

6. **Choice C:** The two quantities are equal.

7. **Choice C:** 56

8. **Choice C:** $0$

- 63 -
9. Choice B: $15x$

10. Choice E: $(6, 2)$

11. Choice A: $\frac{1}{4\pi}$

12. Choice B: $\left(1 + \frac{1}{2}\right) - \left(\frac{1}{21} + \frac{1}{22}\right)$

13. Question 13 is a numeric entry question. In question 13 you were asked to enter either an integer or a decimal. The answer to question 13 is 1.29.

14. The correct answer to question 14 consists of 2 answer choices

   Choice A: 12
   AND
   Choice C: 36

15. Choice A: The tallest male student in the class is 5.8 inches taller than the tallest female student in the class.
1. **Choice A:** 8 to 1

2. **Choice E:** 51%

3. Question 3 is a numeric entry question. In question 3 you were asked to enter a fraction. The answer to question 3 is $\frac{24}{87}$ (or any equivalent fraction).

4. The correct answer to question 4 consists of 6 answer choices.
   - **Choice A:** 2001
   - **Choice B:** 2002
   - **Choice C:** 2003
   - **Choice F:** 2006
   - **Choice G:** 2007
   - **Choice H:** 2008

5. **Choice E:** $640$ million

6. **Choice E:** 140%

7. **Choice A:** $200$ million