Each practice test question in this document is immediately followed by its correct answer and an explanation of the correct answer. If you wish to work through the practice test before consulting the answers and explanations, please use Practice Test #2 first.

Standard timing for each section of the test is shown in the table below:

<table>
<thead>
<tr>
<th>Section Order</th>
<th>Section Name</th>
<th>Standard Time</th>
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</thead>
<tbody>
<tr>
<td>Analytical Writing 1</td>
<td>Analyze an Issue</td>
<td>30 minutes</td>
</tr>
<tr>
<td>Analytical Writing 2</td>
<td>Analyze an Argument</td>
<td>30 minutes</td>
</tr>
<tr>
<td>1</td>
<td>Verbal Reasoning</td>
<td>35 minutes</td>
</tr>
<tr>
<td>2</td>
<td>Verbal Reasoning</td>
<td>35 minutes</td>
</tr>
<tr>
<td>3</td>
<td>Quantitative Reasoning</td>
<td>40 minutes</td>
</tr>
<tr>
<td>4</td>
<td>Quantitative Reasoning</td>
<td>40 minutes</td>
</tr>
</tbody>
</table>

Breaks, including lunch breaks, must occur at the end of sections. Once you complete a section, you may not return to it.

If you are using the large print edition along with another format of the practice test, you may notice some differences in the wording of some questions. Differences in wording between the large print and other editions are the result of adaptations made for each edition.
Instructions for the Verbal Reasoning and Quantitative Reasoning Sections

For your convenience, these instructions are included both in the test book for Sections 1 and 2, and in the test book for Sections 3 and 4. The instructions are the same in both locations.

Important Notes

In the actual test, your scores for the multiple-choice sections will be determined by the number of questions you answer correctly. Nothing is subtracted from a score if you answer a question incorrectly. Therefore, to maximize your scores it is better for you to guess at an answer than not to respond at all. Work as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult for you. Go on to the other questions and come back to the difficult ones later.

Some or all of the passages in this test have been adapted from published material to provide the examinee with significant problems for analysis and evaluation. To make the passages suitable for testing purposes, the style, content, or point of view of the original may have been altered. The ideas contained in the passages do not necessarily represent the opinions of the Graduate Record Examinations Board or Educational Testing Service.

You may use a calculator in the Quantitative Reasoning sections only. You will be provided with a basic calculator and cannot use any other calculator, except as an approved accommodation.
Marking Your Answers

In the actual test, all answers must be marked in the test book. The following instructions describe how answers must be filled in.

Your answers will be hand-scored, so make sure your marks are clear and unambiguous. Examples of acceptable and unacceptable marks will be given with the sample questions.
Question Formats

This practice test may include questions that would not be used in an actual test administered in an alternate format because they have been determined to be less suitable for presentation in such formats.

The questions in these sections have several different formats. A brief description of these formats and instructions for entering your answer choices are given below.

Multiple-Choice Questions—Select One Answer Choice

These standard multiple-choice questions require you to select just one answer choice from a list of options. You will receive credit only if you mark the single correct answer choice and no other.

Example:

What city is the capital of France?

A  Rome
X  Paris
C  London
D  Cairo
<table>
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<tr>
<th></th>
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Unacceptable Marks

If you change an answer, be sure that all previous marks are erased completely. Stray marks and incomplete erasures may be read as intended answers. Blank areas of the test book may be used for working out answers, but do not work out answers near the answer-entry areas. Scratch paper will not be provided, except as an approved accommodation.
Multiple-Choice Questions—Select One or More Answer Choices

Some of these questions specify how many answer choices you must select; others require you to select all that apply. In either case, to receive credit all of the correct answer choices must be marked. These questions are distinguished by the use of a square box to be marked to select an answer choice.

Example:

Select all that apply.

Which of the following countries are in Africa?

- [ ] A. China
- [x] B. Congo
- [ ] C. France
- [x] D. Kenya
Acceptable Marks

A  China
B  Congo
C  France
D  Kenya

Unacceptable Marks

A  China
B  Congo
C  France
D  Kenya
Column Format Questions

This question type presents the answer choices in columns. You must pick one answer choice from each column. You will receive credit only if you mark the correct answer choice in each column.

Example:

Complete the following sentence.
(i) _______ is the capital of (ii)_______.

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<tr>
<th>Blank (i)</th>
<th>Blank (ii)</th>
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<tbody>
<tr>
<td>× Paris</td>
<td>D Canada</td>
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<td>B Rome</td>
<td>× France</td>
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<tr>
<td>C Cairo</td>
<td>F China</td>
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</tbody>
</table>
Numeric-Entry Questions

These questions require a number to be entered by circling entries in a grid. If you are not entering your own answers, your scribe should be familiar with these instructions.

1. Your answer may be an integer, a decimal, or a fraction, and it may be negative.
2. Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Although fractions do not need to be reduced to lowest terms, they may need to be reduced to fit in the grid.
3. Enter the exact answer unless the question asks you to round your answer.
4. If a question asks for a fraction, the grid will have a built-in division slash (/). Otherwise, the grid will have a decimal point.
5. Start your answer in any column, space permitting. Circle no more than one entry in any column of the grid. Columns not needed should be left blank.
6. Write your answer in the boxes at the top of the grid and circle the corresponding entries. You will receive credit only if your grid entries are clearly marked, regardless of the number written in the boxes at the top.
Examples of acceptable ways to use the grid:
Integer answer: 502 (either position is correct)

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Decimal answer:  $-4.13$

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Fraction answer: \(-\frac{2}{10}\)

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</table>
Section 3 follows. In an actual test, your supervisor will tell you when to begin the test.
Section 3
Quantitative Reasoning
25 Questions

Directions: For each question, indicate the best answer using the directions given.

Notes: All numbers used are real numbers.

All figures are assumed to lie in a plane unless otherwise indicated.

Geometric figures, such as lines, circles, triangles, and quadrilaterals, are not necessarily drawn to scale. That is, you should not assume that quantities such as lengths and angle measures are as they appear in a figure. You should assume, however, that lines shown as straight are actually straight, points on a line are in the order shown, and more generally, all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities from how they are drawn in the geometric figure.

Coordinate systems, such as xy-planes and number lines, are drawn to scale; therefore, you can read, estimate, or compare quantities in such figures from how they are drawn in the coordinate system.

Graphical data presentations, such as bar graphs, circle graphs, and line graphs, are drawn to scale; therefore, you can read, estimate, or compare data values from how they are drawn in the graphical data presentation.
For each of Questions 1–9, compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices. A symbol that appears more than once in a question has the same meaning throughout the question.

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
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</table>

Example 1:  \((2)(6)\)  \(2 + 6\)
The correct answer choice for Example 1 is (A). \((2)(6), or 12, is greater than 2 + 6, or 8.\[

Example 2:  \(PS\)  \(SR\)
The correct answer choice is (D). The relationship between \(PS\) and \(SR\) cannot be determined from the information given since equal measures cannot be assumed, even though \(PS\) and \(SR\) appear to be equal in the figure.
O is the center of the circle above.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>5</td>
</tr>
</tbody>
</table>

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.

**Explanation**

In the figure accompanying this question, x is the length of one of the two line segments from the center of the circle to a point inside the circle. In the question you are asked to compare x with 5.

In a circle the easiest line segments to deal with are the radius and the diameter.
In the figure accompanying the question, you can add two radii, each of which “completes” a right triangle, as shown in the figure below.

Since in one of the triangles, the lengths of both legs are known, you can use that triangle to determine the length of the radius of the circle. The triangle has legs of length 3 and 4. If the length of the radius is $r$, then, using the Pythagorean theorem, you get

$$r^2 = 3^2 + 4^2 \text{ or } r^2 = 9 + 16 \text{ or } r^2 = 25,$$

and thus, $r = 5$.

Since the length of the radius of the circle is 5 and the line segment of length $x$ is clearly shorter than the radius, you know that $x < 5$, and the correct answer is Choice B.

You could also notice that the two triangles are congruent, and so $x = 4$, again yielding Choice B.
Runner A ran \( \frac{4}{5} \) kilometer and Runner B ran 800 meters.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>The distance that A ran</td>
<td>The distance that B ran</td>
</tr>
</tbody>
</table>

- **A** Quantity A is greater.
- **B** Quantity B is greater.
- **C** The two quantities are equal.
- **D** The relationship cannot be determined from the information given.

**Explanation**

In this question you are asked to compare two measurements, one given in kilometers and the other in meters. It would be easier to compare these measurements if they were both given in meters or both given in kilometers.

If you choose to convert the distance that Runner B ran from meters to kilometers, you need to use the conversion 1 meter is equal to \( \frac{1}{1,000} \) kilometer. Since B ran 800 meters, it follows that B ran \( (800) \left( \frac{1}{1,000} \right) \), or \( \frac{4}{5} \) kilometer, which is the same distance that A ran.
If you choose to convert the distance that Runner A ran from kilometers to meters, you need to use the conversion 1 kilometer is equal to 1,000 meters. Since A ran $\frac{4}{5}$ kilometer, it follows that A ran $(\frac{4}{5})(1,000)$, or 800 meters, which is the same distance that B ran. Either way, A and B ran the same distance, and the correct answer is Choice C.
\[ x < y < z \]

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{x + y + z}{3} ]</td>
<td>[ y ]</td>
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</table>

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

**Explanation**

In this question you are given that \( x < y < z \), and you are asked to compare \( \frac{x + y + z}{3} \) with \( y \).

Two approaches that you could use to solve this problem are:

**Approach 1:** Search for a mathematical relationship between the two quantities.
**Approach 2:** Plug in numbers for the variables.

**Approach 1:** Note that \( \frac{x + y + z}{3} \) is the average of the three numbers \( x \), \( y \), and \( z \) and that \( y \) is the median. Is the average of 3 numbers always equal to the median? The average could equal the median, but in general they do not have to be equal. Therefore, the correct answer is Choice D.
Approach 2: When you plug in numbers for the variables, it is a good idea to consider what kind of numbers are appropriate to plug in and to choose numbers that are easy to work with, if possible.

Since \( \frac{x + y + z}{3} \) is the average of the three numbers \( x, y, \) and \( z \) and you are comparing it to the median, you may want to try plugging in numbers that are evenly spaced and plugging in numbers that are not evenly spaced.

You can plug in numbers that are both evenly spaced and easy to work with. For example, you can plug in \( x = 1, \ y = 2, \) and \( z = 3. \) In this case, \( \frac{x + y + z}{3} = \frac{1 + 2 + 3}{3} = \frac{6}{3} = 2, \) and so \( \frac{x + y + z}{3} = y. \)

You can also plug in numbers that are not evenly spaced and are easy to work with. For example, you can plug in \( x = 3, \ y = 6, \) and \( z = 12. \) In this case, \( \frac{x + y + z}{3} = \frac{3 + 6 + 12}{3} = \frac{21}{3} = 7, \) and \( \frac{x + y + z}{3} > y. \) Since in the first case, \( \frac{x + y + z}{3} \) is equal to \( y \) and in the second case, it is greater than \( y, \) the relationship between the two quantities \( \frac{x + y + z}{3} \) and \( y \) cannot be determined from the information given. The correct answer is Choice D.
In the figure accompanying this question, $x$ and $y$ are the lengths of the two legs of a triangle, and the leg of length $x$ is opposite a $50^\circ$ angle. In this question you are asked to compare $\frac{x}{y}$ with 1.

One way you can solve this problem is by using the following fact:

**Fact:** If $ABC$ is a triangle and the measure of angle $A$ is greater than the measure of angle $B$, then the side opposite angle $A$ is longer than the side opposite angle $B$. 
Since the third angle of the triangle measures 40°, you can use the fact above to conclude that the side opposite the 50° angle is longer than the side opposite the 40° angle. So \( x > y \) and \( \frac{x}{y} > 1 \), which yields Choice A.

You can also solve this problem without using the fact above. Instead, you can use the strategy of adapting solutions to related problems to determine the relationship between \( x \) and \( y \).

Note that the angles in the 40°–50°–90° triangle in the question differ only a little from the angles in a 45°–45°–90° triangle. How do the lengths of the legs of a 45°–45°–90° triangle compare to the lengths of the legs of the triangle in the question? To make the comparison, add a line segment to the 40°–50°–90° triangle so that the line segment cuts the 50° angle in two parts, making a 45° angle with the horizontal side, as shown in the figure below.

![Diagram of triangle with 45°-45°-90° angles](image)

The 45°–45°–90° triangle has two 45° angles, so \( z = y \), and \( \frac{z}{y} = 1 \). Since \( \frac{z}{y} = 1 \) and \( x > z \), it follows that \( \frac{x}{y} > 1 \). The correct answer is Choice A.
0 < x < y < 1

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – y</td>
<td>y – x</td>
</tr>
</tbody>
</table>

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.

**Explanation**

In this question you are given that 0 < x < y < 1 and you are asked to compare 1 – y with y – x.

Two approaches that you could use to solve this problem are:

**Approach 1**: Translate from algebra to a number line.
**Approach 2**: Plug in values for the variables.
Approach 1: The figure below represents the information given in the problem on a number line.

![Number Line Diagram]

On the number line, 1 – y is the distance between 1 and y, and y – x is the distance between y and x. If y is exactly halfway between x and 1, then 1 – y is equal to y – x; and if y is not halfway between x and 1, then 1 – y is not equal to y – x. But y can be any number between x and 1, so the correct answer is Choice D.

Approach 2: Since this problem involves subtraction, it is a good idea to choose values for x and y that are close to each other as well as values that are far apart. For example, if x = 0.4 and y = 0.5, then 1 – y = 0.5 and y – x = 0.1; and if x = 0.1 and y = 0.9, then 1 – y = 0.1 and y – x = 0.8. This shows that the relationship cannot be determined, and the correct answer is Choice D.
\[ p \text{ is the probability that event } E \text{ will occur, and } s \text{ is the probability that event } E \text{ will not occur.} \]

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p + s )</td>
<td>( ps )</td>
</tr>
</tbody>
</table>

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

**Explanation**

In this question you are given that \( p \) is the probability that event \( E \) will occur, and \( s \) is the probability that event \( E \) will not occur and you are asked to compare \( p + s \) with \( ps \).

Since event \( E \) will either occur or not occur, it follows that \( p + s = 1 \), and the value of Quantity A is always 1. Since Quantity B is the product of the two probabilities \( p \) and \( s \), you need to look at its value for the three cases \( p = 1 \), \( p = 0 \), and \( 0 < p < 1 \).

If \( p = 1 \), then \( s = 0 \); similarly, if \( p = 0 \), then \( s = 1 \). In both cases, \( ps \) is equal to 0.

If \( 0 < p < 1 \), both \( p \) and \( s \) are positive and less than 1, so \( ps \) is positive and less than 1.

Since Quantity A is always equal to 1 and for all three cases Quantity B is less than 1, the correct answer is Choice A.
X is the set of all integers \( n \) that satisfy the inequality \( 2 \leq |n| \leq 5 \).

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. The absolute value of the greatest integer in ( X )</td>
<td>The absolute value of the least integer in ( X )</td>
</tr>
</tbody>
</table>

\( \text{A} \) Quantity \( A \) is greater.

\( \text{B} \) Quantity \( B \) is greater.

\( \text{C} \) The two quantities are equal.

\( \text{D} \) The relationship cannot be determined from the information given.

**Explanation**

In this question it is given that \( X \) is the set of all integers \( n \) that satisfy the inequality \( 2 \leq |n| \leq 5 \), and you are asked to compare the absolute value of the greatest integer in \( X \) with the absolute value of the least integer in \( X \).

When comparing these quantities, it is important to remember that a nonzero number and its negative have the same absolute value. For example, \(|-2| = |2| = 2\). Keeping this in mind, you can see that the positive integers 2, 3, 4, and 5 and the negative integers \(-2, -3, -4, \) and \(-5\) all satisfy the inequalities \( 2 \leq |n| \leq 5 \), and that these are the only such integers. Thus, the set \( X \) consists of the integers \(-5, -4, -3, -2, 2, 3, 4, \) and \( 5 \). The greatest of these integers is \( 5 \), and its absolute value is \( 5 \). The least of these integers is \(-5\), and its absolute value is also \( 5 \). Therefore, Quantity \( A \) is equal to Quantity \( B \). The correct answer is Choice C.
and $m$ are positive numbers, and $m$ is a multiple of 3.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x^m}{x^3}$</td>
<td>$\frac{m}{x^3}$</td>
</tr>
</tbody>
</table>

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

**Explanation**

In this question you are given that $x$ and $m$ are positive numbers, and $m$ is a multiple of 3. You are asked to compare $\frac{x^m}{x^3}$ with $\frac{m}{x^3}$.

Since $\frac{x^m}{x^3} = x^{m-3}$, you need to compare $x^{m-3}$ with $\frac{m}{x^3}$. Since the base in both expressions is the same, a good strategy to use to solve this problem is to plug in numbers for $m$ in both expressions and compare them.
You know that $m$ is a multiple of 3, so the least positive integer you can plug in for $m$ is 3.

If $m = 3$, then $x^{m-3} = 1$ and $x^3 = x$. Since $x$ can be any real number, its relationship to 1 cannot be determined from the information given. This example is sufficient to show that the relationship between $\frac{x^m}{x^3}$ and $x^3$ cannot be determined from the information given. The correct answer is Choice D.
A random variable $Y$ is normally distributed with a mean of 200 and a standard deviation of 10.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>The probability of the event that the value of $Y$ is greater than 220</td>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

A  Quantity A is greater.
B  Quantity B is greater.
C  The two quantities are equal.
D  The relationship cannot be determined from the information given.

**Explanation**

In this question you are given that a random variable $Y$ is normally distributed with mean 200 and standard deviation 10 and you are asked to compare the probability of the event that the value of $Y$ is greater than 220 with $\frac{1}{6}$.

In a normal distribution with mean 200 and standard deviation 10, the value of 210 is 1 standard deviation above the mean, and the value of 220 is 2 standard deviations above the mean. To compare Quantity A with Quantity B, it is not necessary to exactly determine the probability of the event that the value of $Y$ is greater than 220.
Remember that in any normal distribution, almost all of the data values, or about 95% of them, fall within 2 standard deviations on either side of the mean. This means that less than 5% of the values in this distribution will be greater than 220. Thus, the probability of the event that the value of $Y$ is greater than 220 must be less than 5%, or $\frac{1}{20}$, and this is certainly less than $\frac{1}{6}$. The correct answer is Choice B.

Another approach to this problem is to draw a normal curve, or “bell-shaped curve,” that represents the probability distribution of the random variable $Y$, as shown in the figure below.

The curve is symmetric about the mean 200. The values of 210, 220, and 230 are equally spaced to the right of 200 and represent 1, 2, and 3 standard deviations, respectively, above the mean. Similarly,
the values of 190, 180, and 170 are 1, 2, and 3 standard deviations, respectively, below the mean. Quantity A, the probability of the event that the value of \( Y \) is greater than 220, is equal to the area of the shaded region as a fraction of the total area under the curve.

In the figure, the area under the normal curve has been divided into 6 regions and these regions are not equal in area. The shaded region is one of the two smallest of the 6 regions, so its area must be less than \( \frac{1}{6} \) of the total area under the curve. The correct answer is Choice B.
Questions 10–25 have several different formats, including both selecting answers from a list of answer choices and numeric entry. With each question, answer format instructions will be given.

Numeric-Entry Questions

These questions require a number to be entered by circling entries in a grid. If you are not entering in your own answers, your scribe should be familiar with these instructions.

1. Your answer may be an integer, a decimal, or a fraction, and it may be negative.
2. Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Although fractions do not need to be reduced to lowest terms, they may need to be reduced to fit in the grid.
3. Enter the exact answer unless the question asks you to round your answer.
4. If a question asks for a fraction, the grid will have a built-in division slash (/). Otherwise, the grid will have a decimal point.
5. Start your answer in any column, space permitting. Circle no more than one entry in any column of the grid. Columns not needed should be left blank.
6. Write your answer in the boxes at the top of the grid and circle the corresponding entries. You will receive credit only if your grid entries are clearly marked, regardless of the number written in the boxes at the top.
Examples of acceptable ways to use the grid:
Integer answer: 502 (either position is correct)
Decimal answer: $-4.13$

<table>
<thead>
<tr>
<th></th>
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<th>4</th>
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<th>1</th>
<th>3</th>
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<tbody>
<tr>
<td>$\odot$</td>
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</table>
Fraction answer: $\frac{-2}{10}$

<table>
<thead>
<tr>
<th>-</th>
<th>2</th>
<th>/</th>
<th>1</th>
<th>0</th>
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</tbody>
</table>
This question has five answer choices. Select the best one of the answer choices given.

10. The ratio of \( \frac{1}{3} \) to \( \frac{3}{8} \) is equal to the ratio of

A 1 to 8

B 8 to 1

C 8 to 3

D 8 to 9

E 9 to 8

Explanation

In this question you are asked to determine which of the answer choices is equivalent to the ratio \( \frac{1}{3} \) to \( \frac{3}{8} \).

Multiplying both parts of a ratio by the same number produces an equivalent ratio. While you could multiply both fractions in the ratio of \( \frac{1}{3} \) to \( \frac{3}{8} \) by any number, 24 is a good number to choose because it is the least common multiple of 3 and 8. Thus, multiplying both \( \frac{1}{3} \) and \( \frac{3}{8} \) by 24, you get that the ratio of \( \frac{1}{3} \) to \( \frac{3}{8} \) is equal to the ratio of 8 to 9. The correct answer is Choice D, 8 to 9.
An alternate approach to this problem is to express the ratio of \( \frac{1}{3} \) to \( \frac{3}{8} \) as the fraction \( \frac{\frac{1}{3}}{\frac{3}{8}} \). This fraction is equivalent to \( \left( \frac{1}{3} \right) \left( \frac{8}{3} \right) \),
or \( \frac{8}{9} \). The correct answer is Choice D, 8 to 9.
11. A reading list for a humanities course consists of 10 books, of which 4 are biographies and the rest are novels. Each student is required to read a selection of 4 books from the list, including 2 or more biographies. How many selections of 4 books satisfy the requirements?

A  90

B  115

C  130

D  144

E  195

**Explanation**

The requirement to select 4 books, including 2 or more biographies, means that you have to consider three cases. A student can choose 4 biographies and no novels, or 3 biographies and 1 novel, or 2 biographies and 2 novels.

**Case 1:** Choose 4 biographies. This case is easy, as there is only 1 way to choose all four biographies and no novels.

In the other two cases, you have to find the number of ways of choosing the biographies and the number of ways of choosing the novels and then multiply these two numbers.
Case 2: Choose 3 biographies and 1 novel. First, you need to find the number of ways of choosing 3 biographies out of 4. If you think of this as not choosing 1 out of the 4, you see that there are 4 choices. The number of ways of choosing 1 novel out of the 6 novels is 6. Therefore, the total number of choices is \((4)(6) = 24\).

Case 3: Choose 2 biographies and 2 novels. First, you need to find the number of ways of choosing 2 biographies out of 4. This number is sometimes called “4 choose 2” or the number of combinations of 4 objects taken 2 at a time. If you remember the combinations formula, you know that the number of combinations is \(\frac{4!}{2!(4 - 2)!}\) (which is denoted symbolically as \(\binom{4}{2}\) or \(4C_2\)). The value of \(\frac{4!}{2!(4 - 2)!}\) is \(\frac{(4)(3)(2!)}{(2)(2!)} = \frac{(4)(3)}{2} = 6\). Thus, there are 6 ways to choose 2 biographies out of 4. Similarly, the number of ways to choose 2 novels out of 6 is \(\frac{6!}{2!4!} = \frac{(6)(5)}{2} = 15\). Thus, the total number of ways to choose 2 biographies and 2 novels is \((6)(15) = 90\).

Adding the number of ways to choose the books for each of the three cases, you get a total of \(1 + 24 + 90 = 115\). The correct answer is Choice B, 115.
This question does not have any answer choices; it is a numeric entry question. To answer this question, enter a number by circling entries in the grid provided below. The number can include a decimal point, and can be positive, negative, or zero. The number entered cannot be a fraction.

12. In a graduating class of 236 students, 142 took algebra and 121 took chemistry. What is the greatest possible number of students that could have taken both algebra and chemistry?
Explanation

This is the type of problem for which drawing a Venn diagram is usually helpful. The figure below is a Venn diagram you could draw to represent the information given in the question.

Note that the algebra and chemistry numbers given do not separate out the number of students who took both algebra and chemistry, and that this question asks for the greatest possible number of such students. It is a good idea, therefore, to redraw the Venn diagram with the number of students who took both algebra and chemistry separated out. The revised Venn diagram looks like the one in the figure below.
To solve this problem you want the greatest possible value of \( x \). It is clear from the diagram that \( x \) cannot be greater than 142 nor greater than 121, otherwise \( 142 - x \) or \( 121 - x \) would be negative. Hence, \( x \) must be less than or equal to 121. Since there is no information to exclude \( x = 121 \), the correct answer is the number 121.
This question has five answer choices. Select the best one of the answer choices given.

13. In the figure above, if $m \parallel k$ and $s = t + 30$, then $t =$

- A  30
- B  60
- C  75
- D  80
- E  105
Explanation

When trying to solve a geometric problem, it is often helpful to add any known information to the figure. Since corresponding angles have equal measures, you could place two more angle measures on the figure accompanying the question. The figure, with the additional information included, is shown below.

Now, from the figure, you can see that $s + t = 180$. Therefore, since it is given that $s = t + 30$, you can substitute $t + 30$ for $s$ into the equation $s + t = 180$ and get that $(t + 30) + t = 180$, which can be simplified as follows.

\[
(t + 30) + t = 180 \\
2t = 150 \\
t = 75
\]

The correct answer is Choice C, 75.
This question has five answer choices. Select the best one of the answer choices given.

14. If $2x = 3y = 4z = 20$, then $12xyz =$

   - A  16,000
   - B  8,000
   - C  4,000
   - D  800
   - E  10

**Explanation**

In the question you are given that $2x = 3y = 4z = 20$, and you are asked to find the value of $12xyz$. 
One approach you can use to solve this problem is to find the value of all three variables.

\[2x = 20, \text{ or } x = 10\]
\[3y = 20, \text{ or } y = \frac{20}{3}\]
\[4z = 20, \text{ or } z = 5\]

So \(12xyz = 12(10)\left(\frac{20}{3}\right)(5) = 4,000\), and the correct answer is Choice C, 4,000.

Another approach you can use to solve this problem is to notice that \(12xyz = \frac{(2x)(3y)(4z)}{2} = \frac{(20)(20)(20)}{2} = 4,000\). Therefore, the correct answer is Choice C, 4,000.
This question has three answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, or as many as all three of the answer choices.

15. The total amount that Mary paid for a book was equal to the price of the book plus a sales tax that was 4 percent of the price of the book. Mary paid for the book with a $10 bill and received the correct change, which was less than $3.00. Which of the following statements must be true? Indicate all such statements.

A. The price of the book was less than $9.50.
B. The price of the book was greater than $6.90.
C. The sales tax was less than $0.45.

Explanation

For this problem you may find it helpful to translate the given information into an algebraic expression. Since the price of the book is unknown, you can call it $x$ dollars, and then the total amount that Mary paid is $x$ dollars plus 4% of $x$ dollars, or $1.04x$ dollars. The problem states that Mary received some change from a $10 bill, so $1.04x$ dollars must be less than $10. Since the change was less than $3.00, the total amount Mary paid for the book must have been greater than $7.00. You can express this information algebraically by the inequality

$$7.00 < 1.04x < 10.00$$
Solving the inequality for $x$ by dividing by 1.04, and rounding, you get

$$6.73 < x < 9.62$$

So you see that $x$, the price of the book, must be between $6.73$ and $9.62$. With this information, you can quickly examine the first two statements. Choice A, the price of the book was less than $9.50$, is not necessarily true because the price could be as high as $9.61$, and Choice B, the price of the book was greater than $6.90$, is not necessarily true because the price could be as low as $6.74$.

To examine Choice C, the sales tax was less than $0.45$, you could compute the tax for the greatest possible price, which would be 4% of 9.61, or $(0.04)(9.61) = 0.38$. Since this greatest possible tax is less than $0.45$, Choice C must be true.

You can also quickly see that Choice C must be true if you note that 4% of $10.00$ would only be $0.40$, and since the price must be less than $10.00$, the tax must be less than $0.40$. The correct answer consists of one answer choice, Choice C, the sales tax was less than $0.45$. 
16. If \( \frac{1}{211 \cdot 517} \) is expressed as a terminating decimal, how many nonzero digits will the decimal have?

A   One
B   Two
C   Four
D   Six
E   Eleven
Explanation

To convert the fraction $\frac{1}{(2^{11})(5^{17})}$ to a decimal, it is helpful to first write the fraction in powers of 10. Using the rules of exponents, you can write the following.

$$\frac{1}{(2^{11})(5^{17})} = \frac{1}{(2^{11})(5^{11}+6)}$$
$$= \frac{1}{(2^{11})(5^{11})(5^{6})}$$
$$= \frac{1}{(10^{11})(5^{6})}$$
$$= \left(\frac{1}{5}\right)^6 \left(10^{-11}\right)$$
$$= (0.2)^6 \left(10^{-11}\right)$$
$$= \left((2)(10)^{-1}\right)^6 \left(10^{-11}\right)$$
$$= \left(2^6\right)\left(10^{-6}\right)\left(10^{-11}\right)$$
$$= \left(2^6\right)\left(10^{-17}\right)$$
$$= (64)\left(10^{-17}\right)$$

So the decimal has two nonzero digits, 6 and 4. The correct answer is Choice B, two.
Questions 17-20 are based on the data presented on the next page. In order to fit on the page, the data presentation has been turned 90 degrees.
<table>
<thead>
<tr>
<th>Coffee</th>
<th></th>
<th>Amount of Caffeine (milligrams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decaffeinated coffee</td>
<td></td>
<td>150</td>
</tr>
<tr>
<td>Percolated coffee</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Drip-brewed coffee</td>
<td></td>
<td>75</td>
</tr>
<tr>
<td>Instant coffee</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Other beverages</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brewed tea</td>
<td></td>
<td>175</td>
</tr>
<tr>
<td>Instant tea</td>
<td></td>
<td>125</td>
</tr>
<tr>
<td>Cocoa</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Caffeinated soft drinks</td>
<td></td>
<td>75</td>
</tr>
<tr>
<td>Drugs</td>
<td></td>
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<tr>
<td>Weight-loss drugs,</td>
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<td>200</td>
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<tr>
<td>diuretics, and</td>
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<tr>
<td>stimulants</td>
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<td>Pain relievers</td>
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<tr>
<td>Cold/allergy remedies</td>
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</tbody>
</table>

*Based on 5-ounce cups of coffee, tea, and cocoa; 12-ounce cups of soft drinks; and single doses of drugs.

*Source: Food and Drug Administration*
This question has five answer choices. Select the best one of the answer choices given.

17. The least amount of caffeine in a 5-ounce cup of drip-brewed coffee exceeds the greatest amount of caffeine in a 5-ounce cup of cocoa by approximately how many milligrams?

A  160  
B  80  
C  60  
D  40  
E  20  

Explanation

Each horizontal bar in the bar graph shows the possible number of milligrams of caffeine in each of the common beverages and drugs. The least possible amount of caffeine in a 5-ounce cup of drip-brewed coffee is about 60 milligrams, and the greatest possible amount of caffeine in a 5-ounce cup of cocoa is about 20 milligrams. So, the difference is approximately 60 – 20, or 40 milligrams. The correct answer is Choice D, 40.
To check your answer, it is useful to try to solve the problem using another method as well to see if you get the same answer. To solve this problem in another way, note that the distance between each pair of adjacent vertical grid lines represents 25 milligrams of caffeine, and the distance between the high end of the cocoa bar and the low end of the drip-brewed coffee bar is a little more than the distance between a pair of adjacent grid lines. Therefore, the answer is between 25 and 50. Among the choices, only Choice D is between 25 and 50, so the correct answer is Choice D, 40.
This question does not have any answer choices; it is a numeric entry question. To answer this question, enter a number by circling entries in the grid provided below. The number can include a decimal point, and can be positive, negative, or zero. The number entered cannot be a fraction.

18. For how many of the 11 categories of beverages and drugs listed in the graph can the amount of caffeine in the given serving size be less than 50 milligrams?

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</table>
Explanation

In the graph, the left edge of each bar tells you what is the least possible amount of caffeine in the corresponding beverage or drug. A beverage or drug can have less than 50 milligrams of caffeine if the left edge of its bar lies to the left of the vertical line corresponding to 50 milligrams of caffeine. From the graph, you see that there are 9 bars for which this is true. There are only 2 bars that lie entirely to the right of the 50-milligram line—the bar for drip-brewed coffee and the bar for weight-loss drugs, diuretics, and stimulants. So there are 9 categories of beverages and drugs that can have less than 50 milligrams of caffeine in the given serving size. The correct answer is 9.
This question has five answer choices. Select the best one of the answer choices given.

19. Approximately what is the minimum amount of caffeine, in milligrams, consumed per day by a person who daily drinks two 10-ounce mugs of percolated coffee and one 12-ounce cup of a caffeinated soft drink?

A  230  
B  190  
C  140  
D  110  
E  70

**Explanation**

According to the bar graph, the minimum amount of caffeine in a 5-ounce cup of percolated coffee is approximately 40 milligrams. Therefore, the minimum amount of caffeine in two 10-ounce cups of percolated coffee, which is the same as the minimum amount of caffeine in four 5-ounce cups, is approximately \((40)(4)\), or 160 milligrams. The minimum amount of caffeine in a 12-ounce caffeinated soft drink is approximately 30 milligrams. So, the minimum amount of caffeine in two 10-ounce mugs of percolated coffee and one 12-ounce caffeinated soft drink is approximately \(160 + 30\), or 190 milligrams. The correct answer is Choice B, 190.
This question has five answer choices. Select the best one of the answer choices given.

20. Which of the following shows the four types of coffee listed in order according to the range of the amounts of caffeine in a 5-ounce cup, from the least range to the greatest range?

A Decaffeinated, instant, percolated, drip-brewed
B Decaffeinated, instant, drip-brewed, percolated
C Instant, decaffeinated, drip-brewed, percolated
D Instant, drip-brewed, decaffeinated, percolated
E Instant, percolated, drip-brewed, decaffeinated

Explanation

For each of the four types of coffee, the range of the amounts of caffeine is the greatest possible amount minus the least possible amount. In the graph, this difference is represented by the length of the corresponding bar, so you can order the four types of coffee according to the lengths of their corresponding bars, from shortest to longest. From the graph, you can see that the order is decaffeinated coffee, instant coffee, drip-brewed coffee, percolated coffee. The correct answer is Choice B.
This question has five answer choices. Select the best one of the answer choices given.

21. If $s$ is a speed, in miles per hour, at which the energy used per meter during running is twice the energy used per meter during walking, then, according to the graph above, $s$ is between

A 2.5 and 3.0
B 3.0 and 3.5
C 3.5 and 4.0
D 4.0 and 4.5
E 4.5 and 5.0
**Explanation**

The problem asks you to determine the speed at which the energy used per meter during running is twice that used per meter during walking. Graphically, this is the speed for which the running energy is twice as high as the walking energy. The graph indicates that for speeds greater than or equal to 3.0 miles per hour, the running energy is less than twice the walking energy, so the desired speed must be less than 3.0. In fact, the desired speed is between 2.0 (the lowest speed on the graph) and 3.0. There is only one answer choice that is between 2.0 and 3.0; namely, Choice A, which says the desired speed is between 2.5 and 3.0. The correct answer is Choice A.
This question has five answer choices. Select the best one of the answer choices given.

22. If \( n = 2^3 \), then \( n^n = \)

A  \( 2^6 \)
B  \( 2^{11} \)
C  \( 2^{18} \)
D  \( 2^{24} \)
E  \( 2^{27} \)

**Explanation**

In this question you are asked to calculate the value of the expression \( n^n \) when \( n = 2^3 \).

When answering a question in which you are asked to calculate the value of an expression, it is often helpful to look at the answer choices first to see what form they are in. In this question the answer choices are all in the form 2 raised to a power, so you should try to achieve that form. It is given that \( n = 2^3 = 8 \). Therefore,

\[ n^n = (2^3)^8 = 2^{24} \]

The correct answer is Choice D, \( 2^{24} \).
This question has five answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, or as many as all three of the answer choices.

The length of $AB$ is $10\sqrt{3}$.

23. Which of the following statements individually provide(s) sufficient additional information to determine the area of triangle $ABC$ above?

Indicate all such statements.

- **A** $DBC$ is an equilateral triangle.
- **B** $ABD$ is an isosceles triangle.
- **C** The length of $BC$ is equal to the length of $AD$.
- **D** The length of $BC$ is 10.
- **E** The length of $AD$ is 10.

**Explanation**

From the figure accompanying this question you know that $ABC$ is a right triangle with its right angle at vertex $B$. You also know that point $D$ is on the hypotenuse $AC$. You are given that the length of $AB$ is $10\sqrt{3}$. However, because the figure is not necessarily drawn to scale, you don’t know the lengths of $AD$, $DC$, and $BC$. In particular, you don’t know where $D$ is on $AC$. 

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The area of a triangle is \( \frac{1}{2} (\text{base})(\text{height}) \). Thus, the area of right triangle \( ABC \) is equal to \( \frac{1}{2} \) of the length of \( AB \) times the length of \( BC \).

You already know that the length of \( AB \) is \( 10\sqrt{3} \). Any additional information that would allow you to calculate the length of \( BC \) would be sufficient to find the area of triangle \( ABC \). You need to consider each of the five statements individually, as follows.

**Statement A:** \( DBC \) is an equilateral triangle. This statement implies that angle \( DCB \) is a \( 60^\circ \) angle; and therefore, triangle \( ABC \) is a \( 30^\circ-60^\circ-90^\circ \) triangle. Thus the length of \( BC \) can be determined, and this statement provides sufficient additional information to determine the area of triangle \( ABC \).

**Statement B:** \( ABD \) is an isosceles triangle. There is more than one way in which triangle \( ABD \) can be isosceles. Figures 1 and 2 below are two redrawn figures showing triangle \( ABD \) as isosceles. In Figure 1 the length of \( AD \) is equal to the length of \( DB \); and in Figure 2 the length of \( AB \) is equal to the length of \( AD \).

![Figure 1](image-url)
Either of the figures could have been drawn with the length of $BC$ even longer. So, statement B does not provide sufficient additional information to determine the area of triangle $ABC$.

**Statement C:** The length of $BC$ is equal to the length of $AD$. You have no way of finding the length of $AD$ without making other assumptions, so statement C does not provide sufficient additional information to determine the area of triangle $ABC$.

**Statement D:** The length of $BC$ is 10. The length of $BC$ is known, so the area of triangle $ABC$ can be found. Statement D provides sufficient additional information to determine the area of triangle $ABC$.

**Statement E:** The length of $AD$ is 10. The relationship between $AD$ and $BC$ is not known, so statement E does not provide sufficient additional information to determine the area of triangle $ABC$.

Statements A and D individually provide sufficient additional information to determine the area of triangle $ABC$. Therefore, the correct answer consists of two choices A and D; that is, $DBC$ is an equilateral triangle and the length of $BC$ is 10.
This question does not have any answer choices; it is a numeric entry question. To answer this question, enter a number by circling entries in the grid provided below. The number can include a decimal point, and can be positive, negative, or zero. The number entered cannot be a fraction.

\[ a_1, a_2, a_3, \ldots, a_n, \ldots \]

24. In the sequence above, each term after the first term is equal to the preceding term plus the constant \( c \). If \( a_1 + a_3 + a_5 = 27 \), what is the value of \( a_2 + a_4 \) ?

\[
\begin{array}{cccccc}
24 & 24 & 24 & 24 & 24 & 24 \\
\hline
- & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 & 4 & 4 \\
5 & 5 & 5 & 5 & 5 & 5 \\
6 & 6 & 6 & 6 & 6 & 6 \\
7 & 7 & 7 & 7 & 7 & 7 \\
8 & 8 & 8 & 8 & 8 & 8 \\
9 & 9 & 9 & 9 & 9 & 9 \\
\end{array}
\]

**Explanation**

This question gives information about a sequence \( a_1, a_2, a_3, \ldots, a_n, \ldots \) and asks you to calculate the value of \( a_2 + a_4 \).
Note that answering this question requires information only about the first five terms of the sequence. So it is a good idea to work with the relationships among these five terms to see what is happening.

Since you are given that in this sequence each term after $a_1$ is $c$ greater than the previous term, you can rewrite the first five terms of the sequence in terms of $a_1$ and $c$ as follows.

$$a_2 = a_1 + c$$
$$a_3 = a_2 + c = a_1 + 2c$$
$$a_4 = a_1 + 3c$$
$$a_5 = a_1 + 4c$$

From the question, you know that $a_1 + a_3 + a_5 = 27$, and from the equations above,
$$a_1 + a_3 + a_5 = a_1 + (a_1 + 2c) + (a_1 + 4c) = 3a_1 + 6c.$$ So you can conclude that $3a_1 + 6c = 27$, or $a_1 + 2c = 9$.

To find $a_2 + a_4$, you can express $a_2$ and $a_4$ in terms of $a_1$ and $c$ and simplify as follows.

$$a_2 + a_4 = (a_1 + c) + (a_1 + 3c)$$
$$= 2a_1 + 4c$$
$$= 2(a_1 + 2c)$$

But $a_1 + 2c = 9$, so $a_2 + a_4 = 2(9) = 18$. The correct answer is the number 18.
This question has five answer choices. Select the best one of the answer choices given.

25. A desert outpost has a water supply that is sufficient to last 21 days for 15 people. At the same average rate of water consumption per person, how many days would the water supply last for 9 people?

A  28.0
B  32.5
C  35.0
D  37.5
E  42.0

Explanation

The water supply is enough for 15 people to survive 21 days. Assuming the same average rate of water consumption per person, 1 person would have enough water to last for \( (15)(21) = 315 \) days.

Therefore, 9 people would have enough water for \( \frac{1}{9} \) of the 315 days, or 35 days. The correct answer is Choice C.
Another approach to solving this problem is to recognize that the water supply would last \( \frac{15}{9} \) as many days for 9 people as it would for 15 people. Therefore, since the water supply would last 21 days for 15 people, it would last \( \left( \frac{15}{9} \right)(21) \), or 35 days for 9 people. The correct answer is Choice C, 35.0.

End of Section 3 of Revised GRE® Practice Test #2 with Answers and Explanations.
Directions: For each question, indicate the best answer using the directions given.

Notes: All numbers used are real numbers.

All figures are assumed to lie in a plane unless otherwise indicated.

Geometric figures, such as lines, circles, triangles, and quadrilaterals, are not necessarily drawn to scale. That is, you should not assume that quantities such as lengths and angle measures are as they appear in a figure. You should assume, however, that lines shown as straight are actually straight, points on a line are in the order shown, and more generally, all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities from how they are drawn in the geometric figure.

Coordinate systems, such as xy-planes and number lines, are drawn to scale; therefore, you can read, estimate, or compare quantities in such figures from how they are drawn in the coordinate system.

Graphical data presentations, such as bar graphs, circle graphs, and line graphs, are drawn to scale; therefore, you can read, estimate, or compare data values from how they are drawn in the graphical data presentation.
For each of Questions 1–9, compare Quantity A and Quantity B, using additional information centered above the two quantities if such information is given. Select one of the following four answer choices. A symbol that appears more than once in a question has the same meaning throughout the question.

[A] Quantity A is greater.
[B] Quantity B is greater.
[C] The two quantities are equal.
[D] The relationship cannot be determined from the information given.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
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</table>

Example 1: \[(2)(6)\] \[2 + 6\]
The correct answer choice for Example 1 is (A). \((2)(6)\), or 12, is greater than \(2 + 6\), or 8.

Example 2: \[PS\] \[SR\]
The correct answer choice is (D). The relationship between \(PS\) and \(SR\) cannot be determined from the information given since equal measures cannot be assumed, even though \(PS\) and \(SR\) appear to be equal in the figure.
The value of 1 United States dollar is 0.93 Argentine peso and the value of 1 United States dollar is 32.08 Kenyan shillings.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
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<tbody>
<tr>
<td>The dollar value of 1 Argentine peso</td>
<td>The dollar value of 1 Kenyan shilling</td>
</tr>
</tbody>
</table>

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.

**Explanation**

In this question you are given the value of 1 United States dollar in Argentine pesos and in Kenyan shillings, and you are asked to compare the dollar value of 1 Argentine peso with the dollar value of 1 Kenyan shilling. When you are answering Quantitative Comparison questions, it is a good time-saving idea to see whether you can determine the relative sizes of the two quantities being compared without doing any calculations.

Without doing any calculations, you can see from the information given that 1 United States dollar is worth a little less than 1 Argentine peso, so 1 peso is worth more than 1 United States dollar. On the other hand, 1 United States dollar is worth 32.08 Kenyan shillings, so 1 Kenyan shilling is worth only a small fraction of 1 United States dollar. The correct answer is Choice A.
$k$ is a digit in the decimal $1.3k5$, and $1.3k5$ is less than $1.33$.

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<thead>
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<th>Quantity A</th>
<th>Quantity B</th>
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<tbody>
<tr>
<td>$k$</td>
<td>1</td>
</tr>
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</table>

- **A** Quantity A is greater.
- **B** Quantity B is greater.
- **C** The two quantities are equal.
- **D** The relationship cannot be determined from the information given.

**Explanation**

In this question, you are given that $k$ is a digit in the decimal $1.3k5$ and that $1.3k5$ is less than $1.33$, and you are asked to compare $k$ with $1$.

Because $1.3k5$ is less than $1.33$, you can conclude that $1.30 < 1.3k5 < 1.33$. Therefore, $1.3k5$ must equal $1.305$ or $1.315$ or $1.325$, and the digit $k$ must be $0$, $1$, or $2$. The correct answer is Choice D.
$AB$ is a diameter of the circle above.

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<th>Quantity A</th>
<th>Quantity B</th>
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<tbody>
<tr>
<td>3. The length of $AB$</td>
<td>The average (arithmetic mean) of the lengths of $AC$ and $AD$</td>
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</table>

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.
Explanation

In this question you are given a circle and you are asked to compare the length of a diameter of the circle with the average of the lengths of two chords of the circle. Recall that in a circle, any diameter is longer than any other chord that is not a diameter. You are given that $AB$ is a diameter of the circle. It follows that $AC$ and $AD$ are chords that are not diameters, since there is only one diameter with endpoint $A$. Hence, $AB$ is longer than both $AC$ and $AD$. Note that the average of two numbers is always less than or equal to the greater of the two numbers. Therefore, the average of the lengths of $AC$ and $AD$, which is Quantity B, must be less than the length of $AB$, which is Quantity A. The correct answer is Choice A.
\[ st = \sqrt{10} \]

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
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<tbody>
<tr>
<td>( s^2 )</td>
<td>( \frac{10}{t^2} )</td>
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</tbody>
</table>

A) Quantity A is greater.  
B) Quantity B is greater.  
C) The two quantities are equal.  
D) The relationship cannot be determined from the information given.

**Explanation**

In this question you are asked to compare \( s^2 \) with \( \frac{10}{t^2} \). Since it is given that \( st = \sqrt{10} \), it follows that \( (st)^2 = (\sqrt{10})^2 \), and \( s^2 t^2 = 10 \).

Dividing both sides of the equation \( s^2 t^2 = 10 \) by \( t^2 \), you get \( s^2 = \frac{10}{t^2} \). The correct answer is Choice C.

You can look at this problem in another way. You can use the fact that \( st = \sqrt{10} \) to express Quantity A in terms of \( t \). Since \( st = \sqrt{10} \), it follows that \( s = \frac{\sqrt{10}}{t} \), and Quantity A is equal to \( \left( \frac{\sqrt{10}}{t} \right)^2 = \frac{10}{t^2} \), which is the same as Quantity B. The correct answer is Choice C.
Three consecutive integers have a sum of \(-84\).

<table>
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<th>Quantity A</th>
<th>Quantity B</th>
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<tbody>
<tr>
<td>5. The least of the three integers</td>
<td>(-28)</td>
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</tbody>
</table>

A Quantity A is greater.
B Quantity B is greater.
C The two quantities are equal.
D The relationship cannot be determined from the information given.

**Explanation**

In this question it is given that three consecutive integers have a sum of \(-84\). You are asked to compare the least of the three integers with \(-28\).

Two approaches you could use to solve this problem are:

**Approach 1:** Translate from words to algebra.
**Approach 2:** Determine a mathematical relationship between the two quantities.
**Approach 1:** You can represent the least of the three consecutive integers by $x$, and then the three integers would be represented by $x$, $x + 1$, and $x + 2$. It is given that the sum of the three integers is $-84$, so $x + (x + 1) + (x + 2) = -84$. You can solve this equation for $x$ as follows.

\[
x + (x + 1) + (x + 2) = -84 \\
3x + 3 = -84 \\
3x = -87 \\
x = -29
\]

Since the least of the three integers, $-29$, is less than $-28$, the correct answer is Choice B.

**Approach 2:** You could ask yourself what would happen if the least of the three consecutive integers was $-28$. The three consecutive integers would then be $-28$, $-27$, and $-26$, and their sum would be $-81$. But you were given that the sum of the three consecutive integers is $-84$, which is less than $-81$. Therefore, $-28$ is greater than the least of the three consecutive integers, and the correct answer is Choice B.
In the $xy$-plane, the equation of line $k$ is $3x - 2y = 0$.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>The $x$-intercept of line $k$</td>
<td>The $y$-intercept of line $k$</td>
</tr>
</tbody>
</table>

6. Quantity A is greater.

B Quantity B is greater.

C The two quantities are equal.

D The relationship cannot be determined from the information given.

**Explanation**

In this question it is given that the equation of line $k$ in the $xy$-plane is $3x - 2y = 0$. You are asked to compare the $x$-intercept of line $k$ with the $y$-intercept of line $k$.

Two approaches you could use to solve this problem are:

Approach 1: Reason algebraically.

Approach 2: Reason geometrically.

**Approach 1:** To solve this problem algebraically, note that given the equation of a line in the $xy$-plane, the $x$-intercept of the line is the value of $x$ when $y$ equals 0, and the $y$-intercept of the line is the value of $y$ when $x$ equals 0. The equation of line $k$ is $3x - 2y = 0$. If $y = 0$, then $x = 0$; and if $x = 0$, then $y = 0$. Therefore, both the $x$-intercept and $y$-intercept of the line are equal to 0, which means that the line passes through the origin. The correct answer is Choice C.
Approach 2: To solve this problem geometrically, graph the line with equation $3x - 2y = 0$ in the $xy$-plane. Since two points determine a straight line, you can do this by plotting two points on the line and drawing the line they determine. The points $(0, 0)$ and $(2, 3)$ lie on the line, and the graph of the line in the $xy$-plane, with the points $(0, 0)$ and $(2, 3)$ labeled, is shown in the figure below.

The line passes through the origin, and so it crosses both the $x$-axis and the $y$-axis at $(0, 0)$. The correct answer is Choice C.
$n$ is a positive integer that is divisible by 6.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
</tr>
</thead>
<tbody>
<tr>
<td>The remainder when $n$</td>
<td>The remainder when $n$</td>
</tr>
<tr>
<td>is divided by 12</td>
<td>is divided by 18</td>
</tr>
</tbody>
</table>

A. Quantity A is greater.
B. Quantity B is greater.
C. The two quantities are equal.
D. The relationship cannot be determined from the information given.

**Explanation**

In this question it is given that $n$ is a positive integer that is divisible by 6. You are asked to compare the remainder when $n$ is divided by 12 with the remainder when $n$ is divided by 18.

One way to compare the two quantities is to plug in a few values of $n$. If you plug in $n = 36$, you find that both the remainder when $n$ is divided by 12 and the remainder when $n$ is divided by 18 are equal to 0, so Quantity A is equal to Quantity B. However, if you plug in $n = 18$, you find that the remainder when $n$ is divided by 12 is 6 and the remainder when $n$ is divided by 18 is 0, so Quantity B is greater than Quantity A. Therefore, the correct answer is Choice D.
Another way to compare the two quantities is to find all of the possible values of Quantity A and Quantity B. The positive integers that are divisible by 6 are 6, 12, 18, 24, 30, 36, etc. When dividing each of these integers by 12, you get a remainder of either 0 or 6, so Quantity A is either 0 or 6. When dividing each of these integers by 18, you get a remainder of either 0 or 6 or 12, so Quantity B is either 0 or 6 or 12. Note that when the value of Quantity B is 12, the value of Quantity A, 0 or 6, is less than the value of Quantity B; but when the value of Quantity B is 0, the value of Quantity A is greater than or equal to the value of Quantity B. Thus, the correct answer is Choice D.
\[
\frac{1 - x}{x - 1} = \frac{1}{x}
\]

**Quantity A**

8. \(x\)

**Quantity B**

\(-\frac{1}{2}\)

- **A** Quantity A is greater.
- **B** Quantity B is greater.
- **C** The two quantities are equal.
- **D** The relationship cannot be determined from the information given.

**Explanation**

In this question it is given that \(\frac{1 - x}{x - 1} = \frac{1}{x}\) and you are asked to compare \(x\) with \(-\frac{1}{2}\).

One approach you could use to solve this problem is to solve the equation \(\frac{1 - x}{x - 1} = \frac{1}{x}\) for \(x\). Since fractions are defined only when the denominator is not equal to 0, the denominators of both of the fractions in the equation are nonzero. Therefore, \(x \neq 0\) and \(x \neq 1\).

To solve the equation for \(x\), begin by multiplying both sides of the equation by the common denominator \(x(x + 1)\) to get \(x(1 - x) = (x - 1)(1)\). Then proceed as follows.
\[x(1 - x) = (x - 1)(1)\]
\[x - x^2 = x - 1\]
\[x^2 = 1\]

Since \(x^2 = 1\) and \(x \neq 1\), it follows that \(x = -1\).

Quantity A is equal to \(-1\) and Quantity B is equal to \(-\frac{1}{2}\). Therefore, Quantity B is greater, and the correct answer is Choice B.

Another approach is to notice that for all values of \(x \neq 1\), the value of \(\frac{1 - x}{x - 1}\) is equal to \(-1\). You can try plugging in a few numbers for \(x\) to see that this is true. For example, if you plug in \(x = 7\), you get \(\frac{7 - 1}{1 - 7} = \frac{6}{-6} = -1\).

You can also show that for all values of \(x \neq 1\), the value of \(\frac{1 - x}{x - 1}\) is equal to \(-1\) algebraically by rewriting \(1 - x\) as \(-(x - 1)\). Thus,
\[
\frac{1 - x}{x - 1} = \frac{-(x - 1)}{(x - 1)} = -1.
\]
Because the left side of the equation \(\frac{1 - x}{x - 1} = \frac{1}{x}\) is equal to \(-1\), it follows that \(-1 = \frac{1}{x}\), and so \(x = -1\). Therefore, Quantity A is equal to \(-1\), which is less than Quantity B, \(-\frac{1}{2}\), and the correct answer is Choice B.
In a set of 24 positive integers, 12 of the integers are less than 50. The rest are greater than 50.

<table>
<thead>
<tr>
<th>Quantity A</th>
<th>Quantity B</th>
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<td>9. The median of the 24 integers</td>
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**A** Quantity A is greater.
**B** Quantity B is greater.
**C** The two quantities are equal.
**D** The relationship cannot be determined from the information given.

**Explanation**

In this question you are asked to compare the median of 24 integers with 50, given that 12 of the integers are less than 50 and 12 of the integers are greater than 50. In general, the median of a set of \( n \) positive integers, where \( n \) is even, is obtained by ordering the integers from least to greatest and then calculating the average (arithmetic mean) of the two middle integers. For this set of 24 integers, you do not know the values of the two middle integers; you know only that half of the integers are less than 50 and the other half are greater than 50. If the two middle integers in the list are 49 and 51, the median is 50; and if the two middle integers are 49 and 53, the median is 51. Thus the relationship cannot be determined from the information given, and the correct answer is Choice D.
Questions 10–25 have several different formats, including both selecting answers from a list of answer choices and numeric entry. With each question, answer format instructions will be given.

**Numeric-Entry Questions**

These questions require a number to be entered by circling entries in a grid. If you are not entering your own answers, your scribe should be familiar with these instructions.

1. Your answer may be an integer, a decimal, or a fraction, and it may be negative.
2. Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Although fractions do not need to be reduced to lowest terms, they may need to be reduced to fit in the grid.
3. Enter the exact answer unless the question asks you to round your answer.
4. If a question asks for a fraction, the grid will have a built-in division slash (/). Otherwise, the grid will have a decimal point.
5. Start your answer in any column, space permitting. Circle no more than one entry in any column of the grid. Columns not needed should be left blank.
6. Write your answer in the boxes at the top of the grid and circle the corresponding entries. **You will receive credit only if your grid entries are clearly marked, regardless of the number written in the boxes at the top.**
Examples of acceptable ways to use the grid:
Integer answer: 502 (either position is correct)

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Decimal answer: $-4.13$

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Fraction answer: \(-\frac{2}{10}\)

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This question has five answer choices. Select the best one of the answer choices given.

10. The fabric needed to make 3 curtains sells for $8.00 per yard and can be purchased only by the full yard. If the length of fabric required for each curtain is 1.6 yards and all of the fabric is purchased as a single length, what is the total cost of the fabric that needs to be purchased for the 3 curtains?

   A   $40.00
   B   $38.40
   C   $24.00
   D   $16.00
   E   $12.80

**Explanation**

Since 1.6 yards of fabric are required for each curtain, it follows that (3)(1.6), or 4.8, yards of fabric are required to make the 3 curtains. The fabric can be purchased only by the full yard, so 5 yards of fabric would need to be purchased. Since the fabric sells for $8.00 per yard, the total cost of the fabric is $40.00. The correct answer is Choice A, $40.00.
This question has three answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, and as many as all three of the answer choices.

11. In the $xy$-plane, line $k$ is a line that does not pass through the origin.

Which of the following statements individually provide(s) sufficient additional information to determine whether the slope of line $k$ is negative?

Indicate all such statements.

A  The $x$-intercept of line $k$ is twice the $y$-intercept of line $k$.

B  The product of the $x$-intercept and the $y$-intercept of line $k$ is positive.

C  Line $k$ passes through the points $(a, b)$ and $(r, s)$, where $(a - r)(b - s) < 0$.

Explanation

For questions involving a coordinate system, it is often helpful to draw a figure representing the problem situation. The problem situation in this question involves determining when lines that do not pass through the origin have a negative slope. You can begin to solve this problem by drawing some lines with negative slopes in the $xy$-plane, such as those in the figure below.
From the figure you can see that for each line that does not pass through the origin, the *x* - and *y*-intercepts are either both positive or both negative. Conversely, you can see that if the *x*- and *y*-intercepts of a line have the same sign then the slope of the line is negative.

You can use this fact to examine the information given in the first two statements. Remember that you need to evaluate each statement by itself.

Choice A states that the *x*-intercept is twice the *y*-intercept, so you can conclude that both intercepts have the same sign, and thus the slope of line *k* is negative. So the information in Choice A is sufficient to determine that the slope of line *k* is negative.
Choice B states that the product of the \(x\)-intercept and the \(y\)-intercept is positive. You know that the product of two numbers is positive if both factors have the same sign. So this information is also sufficient to determine that the slope of line \(k\) is negative.

To evaluate Choice C, it is helpful to recall the definition of the slope of a line passing through two given points. You may remember it as “rise over run.” If the two points are \((a, b)\) and \((r, s)\), then the slope is \(\frac{b - s}{a - r}\).

Choice C states that the product of the quantities \((a - r)\) and \((b - s)\) is negative. Note that these are the denominator and the numerator, respectively, of \(\frac{b - s}{a - r}\), the slope of line \(k\). So you can conclude that \((a - r)\) and \((b - s)\) have opposite signs and the slope of line \(k\) is negative. The information in Choice C is sufficient to determine that the slope of line \(k\) is negative.

So each of the three statements individually provides sufficient information to determine whether the slope of line \(k\) is negative. The correct answer consists of Choices A, B, and C; that is, the \(x\)-intercept of line \(k\) is twice the \(y\)-intercept of line \(k\), the product of the \(x\)-intercept and the \(y\)-intercept of line \(k\) is positive, and line \(k\) passes through the points \((a, b)\) and \((r, s)\), where \((a - r)(b - s) < 0\).
This question has five answer choices. Select the best one of the answer choices given.

The distance from Centerville to a freight train is given by the expression $-10t + 115$, and the distance from Centerville to a passenger train is given by the expression $-20t + 150$.

12. The expressions above give the distance from Centerville to each of two trains $t$ hours after 12:00 noon. At what time after 12:00 noon will the trains be equidistant from Centerville?

- [A] 1:30
- [B] 3:30
- [C] 5:10
- [D] 8:50
- [E] 11:30

**Explanation**

The distance between the freight train and Centerville at $t$ hours past noon is $-10t + 115$. The distance between the passenger train and Centerville at $t$ hours past noon is $-20t + 150$. To find out at what time the distances will be the same you need to equate the two expressions and solve for $t$ as follows.

\[
-10t + 115 = -20t + 150 \\
10t + 115 = 150 \\
10t = 35 \\
t = 3.5
\]

Therefore, the two trains will be the same distance from Centerville at 3.5 hours past noon, or at 3:30. The correct answer is Choice B, 3:30.
13. The company at which Mark is employed has 80 employees, each of whom has a different salary. Mark’s salary of $43,700 is the second-highest salary in the first quartile of the 80 salaries. If the company were to hire 8 new employees at salaries that are less than the lowest of the 80 salaries, what would Mark’s salary be with respect to the quartiles of the 88 salaries at the company, assuming no other changes in the salaries?

A  The fourth-highest salary in the first quartile
B  The highest salary in the first quartile
C  The second-lowest salary in the second quartile
D  The third-lowest salary in the second quartile
E  The fifth-lowest salary in the second quartile
**Explanation**

In this question you are told that Mark’s salary is the second-highest in the first quartile. From this you can conclude that the word *quartile* refers to one of the four groups that are created by listing the data in increasing order and then dividing the data into four groups of equal size. When the salaries of the 80 employees are listed in order, the 20 lowest salaries (that is, the salaries in the first quartile) are the first 20 salaries in the list. Since Mark’s salary is the second-highest in the first quartile, 18 salaries in that quartile are lower than his, and one salary in that quartile is higher than his. After the salaries of the 8 new employees are added, there are 26 salaries that are lower than Mark’s. The lowest 22 of those would be in the first quartile of the 88 salaries, and the remaining 4 (salaries 23 to 26) would be in the second quartile, followed by Mark’s salary. This puts Mark at the fifth-lowest salary in the second quartile. The correct answer is Choice E.

Another way to approach this problem is to think of all 80 salaries numbered in order from least to greatest, the lowest salary at the number 1 position and the greatest salary at the number 80 position. There are 20 positions in each quartile, and Mark’s salary is at position 19. The table below shows the salary positions and the quartile into which each position falls.
<table>
<thead>
<tr>
<th>First quartile</th>
<th>Second quartile</th>
<th>Third quartile</th>
<th>Fourth quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>41</td>
<td>61</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>42</td>
<td>62</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
<td>43</td>
<td>63</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>18</td>
<td>38</td>
<td>58</td>
<td>78</td>
</tr>
<tr>
<td>19 Mark’s salary</td>
<td>39</td>
<td>59</td>
<td>79</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
</tr>
</tbody>
</table>
Position 19, where Mark’s salary appears, is second-highest in the first quartile.

To find what Mark’s position is with respect to the quartiles of the 88 salaries, you need to add the 8 new salaries to the list, renumber the list from 1 to 88, and put 22 salaries in each quartile. Because the 8 new salaries are less than the original 80 salaries, they must be listed in positions 1 through 8, and all salaries in the original list must move up by 8 positions in the renumbered list. In particular, Mark’s salary moves from position 19 to position 27. The table below shows the renumbered list. Mark’s salary is in position 27, the fifth position in the second quartile.
<table>
<thead>
<tr>
<th>First quartile</th>
<th>Second quartile</th>
<th>Third quartile</th>
<th>Fourth quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23</td>
<td>45</td>
<td>67</td>
</tr>
<tr>
<td>2</td>
<td>New salaries</td>
<td>24</td>
<td>46</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>47</td>
<td>68</td>
</tr>
<tr>
<td>9</td>
<td>Salary at</td>
<td>26</td>
<td>48</td>
</tr>
<tr>
<td>20</td>
<td>position 1</td>
<td>27 Mark’s</td>
<td>49</td>
</tr>
<tr>
<td>21</td>
<td>of original</td>
<td>42</td>
<td>64</td>
</tr>
<tr>
<td>22</td>
<td>list</td>
<td>43</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>44</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>86</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>87</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>88</td>
</tr>
</tbody>
</table>
Since Mark’s salary is in the fifth position in the second quartile and the salaries are listed in order from least to greatest, Mark’s salary would be the fifth-lowest in the second quartile. The correct answer is Choice E, the fifth-lowest salary in the second quartile.
This question does not have any answer choices; it is a numeric-entry question. To answer this question, enter a number by circling entries in the grid provided below. The number can include a decimal point, and can be positive, negative, or zero. The number entered cannot be a fraction.

14. In the xy-plane, the point with coordinates (−6, −7) is the center of circle C. The point with coordinates (−6, 5) lies inside C, and the point with coordinates (8, −7) lies outside C. If m is the radius of C and m is an integer, what is the value of m?

\[ m = \begin{array}{cccccc}
- & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 & 4 & 4 \\
5 & 5 & 5 & 5 & 5 & 5 \\
6 & 6 & 6 & 6 & 6 & 6 \\
7 & 7 & 7 & 7 & 7 & 7 \\
8 & 8 & 8 & 8 & 8 & 8 \\
9 & 9 & 9 & 9 & 9 & 9 \\
\end{array} \]
**Explanation**

A strategy that is often helpful in working with geometry problems is drawing a figure that represents the given information as accurately as possible.

In this question you are given that the point with coordinates $(−6, 7)$ is the center of circle $C$, the point with coordinates $(−6, 5)$ lies inside circle $C$, and the point with coordinates $(8, −7)$ lies outside circle $C$, so you could draw a circle in the $xy$-plane, as the one shown in the figure below.
From the figure, you can conclude that the distance between (−6, −7) and (−6, 5) is $7 + 5$, or 12, and the radius of $C$ must be greater than 12. You can also conclude that the distance between (−6, −7) and (8, −7) is $6 + 8$, or 14, and the radius of $C$ must be less than 14. Therefore, since the radius is an integer greater than 12 and less than 14, it must be 13. The correct answer is 13.
This question has five answer choices. Select the best one of the answer choices given.

15. If \( \frac{-m}{19} \) is an even integer, which of the following must be true?
   
   A. \( m \) is a negative number.
   
   B. \( m \) is a positive number.
   
   C. \( m \) is a prime number.
   
   D. \( m \) is an odd integer.
   
   E. \( m \) is an even integer.

Explanation

An even integer is a multiple of 2. If \( \frac{-m}{19} \) is an even integer, it must equal 2 times some integer \( k \). This means that \( \frac{-m}{19} = 2k \), or \( m = -19(2k) = 2(-19k) \), which is a multiple of 2. Thus \( m \) must be an even integer, and the correct answer is Choice E, \( m \) is an even integer. You can see that none of the other choices can be the correct answer by evaluating them as follows.

(A) \( m \) does not have to be a negative number for \( \frac{-m}{19} \) to be even.

For example, if \( m = 38 \), then \( \frac{-m}{19} = -2 \), which is an even number.
(B) $m$ does not have to be a positive number for $-\frac{m}{19}$ to be even.

For example, if $m = -38$, then $-\frac{m}{19} = 2$, which is an even number.

(C) The number used in (A), $m = 38$, shows that $m$ does not have to be a prime number. In fact, because $m$ is the product of at least two prime numbers (2 and 19), $m$ cannot be a prime number.

(D) Since $m$ must be an even integer, $m$ cannot be an odd integer.
This question has three answer choices. Select all the answer choices that apply. The correct answer to a question of this type could consist of as few as one, or as many as all three of the answer choices.

16. The integer \( v \) is greater than 1. If \( v \) is the square of an integer, which of the following numbers must also be the square of an integer? 
Indicate all such numbers.

\[
\begin{align*}
A & : 81v \\
B & : 25v + 10\sqrt{v} + 1 \\
C & : 4v^2 + 4\sqrt{v} + 1 \\
\end{align*}
\]

**Explanation**

If \( v \) is the square of an integer, then \( \sqrt{v} \) is an integer. You can use this fact, together with the fact that the product and the sum of integers are also integers, to examine the first two choices.

Choice A: The square root of \( 81v \) is \( 9\sqrt{v} \), which is an integer. So \( 81v \) is the square of an integer.

Choice B: \( 25v + 10\sqrt{v} + 1 = (5\sqrt{v} + 1)^2 \) and \( 5\sqrt{v} + 1 \) is an integer. So \( 25v + 10\sqrt{v} + 1 \) is the square of an integer.
Choice C: Since there is no obvious way to factor the given expression, you may suspect that it is not the square of an integer. To show that a given statement is not true, it is sufficient to find one counterexample. In this case, you need to find one value of \( v \) such that \( v \) is the square of an integer but \( 4v^2 + 4\sqrt{v} + 1 \) is not the square of an integer. If \( v = 4 \), then \( 4v^2 + 4\sqrt{v} + 1 = 64 + 8 + 1 = 73 \), which is not the square of an integer. This proves that \( 4v^2 + 4\sqrt{v} + 1 \) does not have to be the square of an integer.

The correct answer consists of two choices: A and B; that is \( 81v \) and \( 25v + 10\sqrt{v} + 1 \).
Questions 17-20 are based on the data presented on this and the next page.

DISTANCE TRAVELED BY A CAR ACCORDING TO THE CAR’S SPEED WHEN THE DRIVER IS SIGNALED TO STOP

Distance Traveled During Reaction Time*

<table>
<thead>
<tr>
<th>Speed (miles per hour)</th>
<th>Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>
Distance Traveled After Brakes Have Been Applied

*Reaction time is the time period that begins when the driver is signaled to stop and ends when the driver applies the brakes.

Note: Total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after brakes have been applied.
17. The speed, in miles per hour, at which the car travels a distance of 52 feet during reaction time is closest to which of the following?

- A 43
- B 47
- C 51
- D 55
- E 59

**Explanation**

It is a good idea to look at the graphs before you try to answer the questions, so you can become familiar with the information contained in the graphs. Then, as you read each question, you should think about which of the graphs contains the information you need to solve the problem. It could be that all the information you need to solve the problem is contained in one of the graphs, or it could be that you need to get information from both of the graphs.

The first graph shows the relationship between the speed of the automobile and the distance it traveled during the reaction time. Therefore, the answer to this question is found using this graph by reading the speed, in miles per hour, corresponding to a distance of 52. A distance of 52 feet is a little above the distance of 50 feet on the vertical axis of the graph. On the graph, the speed corresponding to a distance of 52 feet is a little less than 50 miles per hour. The correct answer is Choice B, 47.
This question has five answer choices. Select the best one of the answer choices given.

18. Approximately what is the total stopping distance, in feet, if the car is traveling at a speed of 40 miles per hour when the driver is signaled to stop?

- A  130
- B  110
- C  90
- D  70
- E  40

**Explanation**

Since the total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after the brakes have been applied, you need information from both graphs to answer this question. At a speed of 40 miles per hour, the distance traveled during reaction time is a little less than 45 feet, and the distance traveled after the brakes have been applied is 88 feet. Since $45 + 88 = 133$, the correct answer is Choice A, 130.
This question has five answer choices. Select the best one of the answer choices given.

19. Of the following, which is the greatest speed, in miles per hour, at which the car can travel and stop with a total stopping distance of less than 200 feet?

   A  50
   B  55
   C  60
   D  65
   E  70
Explanation

Since the total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after the brakes have been applied, you need information from both graphs to answer this question. A good strategy for solving this problem is to calculate the total stopping distance for the speeds given in the answer choices. For a speed of 50 miles per hour, the distance traveled during reaction time is about 55 feet, and the distance traveled after the brakes have been applied is 137 feet; therefore, the total stopping distance is about $55 + 137$, or 192 feet. For a speed of 55 miles per hour, the distance traveled during reaction time is about 60 feet, and the distance traveled after the brakes have been applied is about 170 feet; therefore, the total stopping distance is about $60 + 170$, or 230 feet. Since the speeds in the remaining choices are greater than 55 miles per hour and both types of stopping distances increase as the speed increases, it follows that the total stopping distances for all the remaining choices are greater than 200 feet. The correct answer is Choice A, 50.
This question has five answer choices. Select the best one of the answer choices given.

20. The total stopping distance for the car traveling at 60 miles per hour is approximately what percent greater than the total stopping distance for the car traveling at 50 miles per hour?

A   22%
B   30%
C   38%
D   45%
E   52%

**Explanation**

To solve this problem you need to find the total stopping distance at 50 miles per hour and at 60 miles per hour, find their difference, and then express the difference as a percent of the shorter total stopping distance. You need to use both graphs to find the total stopping distances. At 50 miles per hour, the total stopping distance is approximately $55 + 137 = 192$ feet; and at 60 miles per hour it is approximately $66 + 198 = 264$ feet. The difference of 72 feet as a percent of 192 feet is $\frac{72}{192} = 0.375$, or approximately 38%.

The correct answer is Choice C, 38%.
This question has five answer choices. Select the best one of the answer choices given.

21. What is the least positive integer that is not a factor of 25! and is not a prime number?

   A  26
   B  28
   C  36
   D  56
   E  58

**Explanation**

In this question you are asked what is the least positive integer that is not a factor of 25! and is not a prime number.

Note that 25! is equal to the product of all positive integers from 1 to 25, inclusive. Thus, every positive integer less than or equal to 25 is a factor of 25!. Also, any integer greater than 25 that can be expressed as the product of different positive integers less than 25 is a factor of 25!. In view of this, it’s reasonable to consider the next few integers greater than 25, including answer choices A and B.
Choice A, 26, is equal to (2)(13). Both 2 and 13 are factors of 25!, so 26 is also a factor of 25!. The same is true for 27, or (3)(9), and for Choice B, 28, or (4)(7). However, the next integer, 29, is a prime number greater than 25, and as such, it has no positive factors (other than 1) that are less than or equal to 25. Therefore, 29 is the least positive integer that is not a factor of 25!. However, the question asks for an integer that is not a prime number, so 29 is not the answer.

At this point, you could consider 30, 31, 32, etc., but it is quicker to look at the rest of the choices. Choice C, 36, is equal to (4)(9). Both 4 and 9 are factors of 25!, so 36 is also a factor of 25!. Choice D, 56, is equal to (4)(14). Both 4 and 14 are factors of 25!, so 56 is also a factor of 25!. Choice E, 58, is equal to (2)(29). Although 2 is a factor of 25!, the prime number 29, as noted earlier, is not a factor of 25!, and therefore 58 is not a factor of 25!. The correct answer must be Choice E, 58.

The explanation above uses a process of elimination to arrive at Choice E, which is sometimes the most efficient way to find the correct answer. However, one can also show directly that the correct answer is 58. For if a positive integer \( n \) is not a factor of 25!, then one of the following must be true:

1. \( n \) is a prime number greater than 25, like 29 or 31, or a multiple of such a prime number, like 58 or 62;
2. \( n \) is so great a multiple of some prime number less than 25, that it must be greater than 58.
To see that (1) or (2) is true, recall that every integer greater than 1 has a unique prime factorization, and consider the prime factorization of $25!$. The prime factors of $25!$ are 2, 3, 5, 7, 11, 13, 17, 19, and 23, some of which occur more than once in the product $25!$. For example, there are 8 positive multiples of 3 less than 25, namely 3, 6, 9, 12, 15, 18, 21, and 24. The prime number 3 occurs once in each of these multiples, except for 9 and 18, in which it occurs twice. Thus, the factor 3 occurs 10 times in the prime factorization of $25!$. The same reasoning can be used to find the number of times that each of the prime factors occur, yielding the prime factorization

\[ 25! = \left(2^{22}\right)\left(3^{10}\right)\left(5^6\right)\left(7^3\right)\left(11^2\right)(13)(17)(19)(23). \]

Any integer whose prime factorization is a combination of one or more of the factors in the prime factorization of $25!$, perhaps with lesser exponents, is a factor of $25!$. Equivalently, if the positive integer $n$ is not a factor of $25!$, then, restating (1) and (2) above, the prime factorization of $n$ must

(1) include a prime number greater than 25; or
(2) have a greater exponent for one of the prime numbers in the prime factorization of $25!$.

For (2), the least possibilities are $2^{23}$, $3^{11}$, $5^{7}$, $7^{4}$, $11^{3}$, $13^{2}$, $17^{2}$, $19^{2}$, and $23^{2}$. Clearly, all of these are greater than 58. The least possibility for (1) that is not a prime number is 58, and the least possibility for (2) is greater than 58, so Choice E, 58, is the correct answer.
This question has five answer choices. Select the best one of the answer choices given.

22. If $0 < a < 1 < b$, which of the following is true about the reciprocals of $a$ and $b$?

A $1 < \frac{1}{a} < \frac{1}{b}$

B $\frac{1}{a} < 1 < \frac{1}{b}$

C $\frac{1}{a} < \frac{1}{b} < 1$

D $\frac{1}{b} < 1 < \frac{1}{a}$

E $\frac{1}{b} < \frac{1}{a} < 1$
Explanation

In this question it is given that \( 0 < a < 1 < b \). The question asks which of the answer choices is a true statement about the reciprocals of \( a \) and \( b \).

To answer this question, you must first look at the answer choices. Note that all of the choices are possible orderings of the quantities \( \frac{1}{a} \), \( \frac{1}{b} \), and 1 from least to greatest. So to answer the question you must put the three quantities in order from least to greatest. The inequality \( 0 < a < 1 < b \) tells you that \( 0 < a < 1 \) and that \( b > 1 \). Since \( a \) is a value between 0 and 1, the value of \( \frac{1}{a} \) must be greater than 1. Since \( b \) is greater than 1, the value of \( \frac{1}{b} \) must be less than 1. So you know that \( \frac{1}{a} > 1 \) and that \( \frac{1}{b} < 1 \), or combined in one expression, \( \frac{1}{b} < 1 < \frac{1}{a} \), and the correct answer is Choice D, \( \frac{1}{b} < 1 < \frac{1}{a} \).
This question has five answer choices. Select the best one of the answer choices given.

23. In the figure above, O and P are the centers of the two circles. If each circle has radius $r$, what is the area of the shaded region?

A $\frac{\sqrt{2}}{2} r^2$
B $\frac{\sqrt{3}}{2} r^2$
C $\sqrt{2} r^2$
D $\sqrt{3} r^2$
E $2\sqrt{3} r^2$

**Explanation**

If a geometric problem contains a figure, it can be helpful to draw additional lines and add information given in the text of the problem.
to the figure. For circles, the helpful additional lines are often radii or diameters. In this case, drawing radius \( OP \) will divide the shaded region into two triangles, as shown in the figure below.

Circle \( O \) and circle \( P \) have the same radius, \( r \). Therefore, in each of the triangles, all three sides have length \( r \), and each of the triangles is equilateral. If you remember from geometry that the height of an equilateral triangle with sides of length \( r \) is \( \frac{\sqrt{3}}{2} r \), you could use that fact in solving the problem. However, if you do not remember what the height is, you can use the figure below to help you find the height.
Using the Pythagorean theorem, you get

\[
\left(\frac{r}{2}\right)^2 + h^2 = r^2
\]

\[
\frac{r^2}{4} + h^2 = r^2
\]

\[
h^2 = \frac{3}{4}r^2
\]

\[
h = \frac{\sqrt{3}}{2}r
\]

So the area of the equilateral triangle is

\[
\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(r)\left(\frac{\sqrt{3}}{2}r\right) = \frac{\sqrt{3}}{4}r^2
\]

Since the shaded region consists of 2 equilateral triangles with sides of length \(r\), the area of the shaded region is

\[
2\left(\frac{\sqrt{3}}{4}r^2\right) = \frac{\sqrt{3}}{2}r^2
\]

and the correct answer is Choice B, \(\frac{\sqrt{3}}{2}r^2\). 
This question does not have any answer choices; it is a numeric entry question. To answer this question enter a fraction by circling entries in the grid provided below. The fraction can be positive or negative. Neither the numerator nor the denominator of the fraction can include a decimal point. The fraction does not have to be in lowest terms.

24. Of the 20 lightbulbs in a box, 2 are defective. An inspector will select 2 lightbulbs simultaneously and at random from the box. What is the probability that neither of the lightbulbs selected will be defective?

Give your answer as a fraction.
Explanation

The desired probability corresponds to the fraction

\[
\frac{\text{the number of ways that 2 lightbulbs, both of which are not defective, can be chosen}}{\text{the number of ways that 2 lightbulbs can be chosen}}
\]

In order to calculate the desired probability, you need to calculate the values of the numerator and the denominator of this fraction.

In the box there are 20 lightbulbs, 18 of which are not defective. The numerator of the fraction is the number of ways that 2 lightbulbs can be chosen from the 18 that are not defective, also known as the number of combinations of 18 objects taken 2 at a time.

If you remember the combinations formula, you know that the number of combinations is \( \frac{18!}{2!(18 - 2)!} \) (which is denoted symbolically as \( \binom{18}{2} \) or \( 18 \binom{2} \)). Simplifying, you get

\[
\frac{18!}{2!16!} = \frac{(18)(17)(16)!}{(2)(16)!} = \frac{(18)(17)}{2} = 153
\]

Similarly, the denominator of the fraction is the number of ways that 2 lightbulbs can be chosen from the 20 in the box, which is
Therefore, the probability that neither of the lightbulbs selected will be defective is \( \frac{153}{190} \). The correct answer is \( \frac{153}{190} \).

Another approach is to look at the selection of the two lightbulbs separately. The problem states that lightbulbs are selected simultaneously. However, the timing of the selection only ensures that the same lightbulb is not chosen twice. This is equivalent to choosing one lightbulb first and then choosing a second lightbulb without replacing the first. The probability that the first lightbulb selected will not be defective is \( \frac{18}{20} \). If the first lightbulb selected is not defective, there will be 19 lightbulbs left to choose from, 17 of which are not defective. Thus, the probability that the second lightbulb selected will not be defective is \( \frac{17}{19} \). The probability that both lightbulbs selected will not be defective is the product of these two probabilities. Thus, the desired probability is \( \left( \frac{18}{20} \right) \left( \frac{17}{19} \right) = \frac{153}{190} \). The correct answer is \( \frac{153}{190} \).
This question has five answer choices. Select the best one of the answer choices given.

25. What is the perimeter, in meters, of a rectangular playground 24 meters wide that has the same area as a rectangular playground 64 meters long and 48 meters wide?

   A  112  
   B  152  
   C  224  
   D  256  
   E  304

Explanation

The area of the rectangular playground that is 64 meters long and 48 meters wide is \((64)(48) = 3,072\) square meters. The second playground, which has the same area, is 24 meters wide and \(\frac{3,072}{24} = 128\) meters long. Therefore, the perimeter of the second playground is \((2)(24) + (2)(128) = 304\) meters. The correct answer is Choice E, 304.

End of Section 4 of Revised GRE® Practice Test # 2 with Answers and Explanations

This is the end of Revised GRE® Practice Test # 2 with Answers and Explanations.